

Math 220 AD9 Spring 2009 Worksheet 3

1. Without using your calculator, find the exact values of:

(a) $\ln(e^{\sqrt{2}})$

(b) $\log_6\left(\frac{1}{36}\right)$

2. Solve each equation for x :

(a) $\ln(2x - 1) = 3$

(b) $e^{e^x} = 4$

(c) $\log_x 32 = 5$

(d) $\ln(x^2 - \pi^2) - \ln(x + \pi) = 7$ (also find the domain of $\ln(x^2 - \pi^2) - \ln(x + \pi)$.)

3. The Richter magnitude M of an earthquake is defined in terms of the energy E in Joules released by the earthquake, with $\log_1 0E = 4.4 + 1.5M$. Find the energy for earthquakes with magnitudes 4, 5, and 6, respectively. For each increase in M of 1, by what factor does E change?

4. Without using your calculator, estimate $\log_4 \frac{1}{10}$, $\log_4 60$, $\log_3 7$.

5. Simplify

$$\frac{2^3 \cdot 7^{3/2} \cdot 5^{1/2}}{2^5 \cdot \sqrt{175} \cdot 8^{1/3}}$$

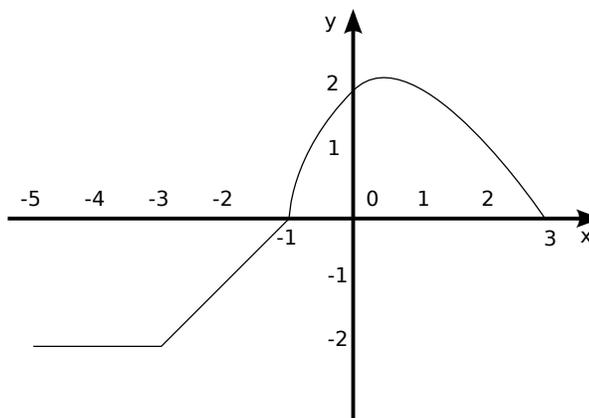
6. Let $g(z) = z^2 + 1$ and $h(z) = 2z - 1$. Find the compositions $(g \circ h)(z)$ and $(h \circ g)(z)$.

7. Let $f(x) = x^3$ and $g(x) = \cos x$. Find the compositions $(f \circ g)(x)$ and $(g \circ f)(x)$. Are the composite functions equal? Why or why not?

8. Let $f(x) = x^2$ and $g(x) = \sqrt{x}$. Find the composites $(f \circ g)(x)$ and $(g \circ f)(x)$. Are these composite functions equal?

9. Let $h(x) = (x^2 + 4)^{3/2}$. Find functions f and g such that $h(x) = f(g(x))$. Is there more than one possibility for f and g ?

10. Let $f(x)$ be the function graphed below. First explain in words how to obtain the graphs of the functions listed below and then draw rough sketches of these graphs.



- (a) $-f(x)$ (b) $f(-x)$
 (c) $2f(x)$ (d) $f(2x)$
 (e) $f(x+3)$ (f) $f(x)+3$
 (g) $f(2x-6)$ (h) $f\left(\frac{x}{2}\right)-6$

11. Explain in words how to obtain the graph of $y = 2x^2 - 4x + 3$ from the graph of $y = x^2$.
12. For each of the following, find a function of the form $y = Ca^x$ that passes through the supplied points.
 - (a) $(0, 2)$ and $(1, 4)$.
 - (b) $(-1, 2)$ and $(0, 1)$.
 - (c) $(-2, 0.0625)$ and $(2, 16)$.
13. If you had two functions a^x and b^x , a and b both positive, what information would you need to determine which was larger for *negative* values of x ? How little information about a and b can we get away with and still be able to answer the question?
14. Sketch a graph of the function $f(x) = 2^x$, labeling a few points. For positive x values, which function seems to be increasing the most quickly: 2^x , x^2 , or $x^2 0$?
15. Sketch graphs of 2^x , 5^x , 1^x , and $(1/2)^x$ on the same axes. Label the y-intercepts. For positive x , which is largest? Which functions are increasing everywhere? Which functions are decreasing everywhere?
16. It's known that for all positive numbers a , $0^a = 0$. It's also true that for all positive numbers b , $b^0 = 1$. Given those two facts, what value do you think 0^0 should take? What does the graph of x^x look like near zero?
17. Find all possible solutions of the following equations.
 - (a) $\sin^2 x + 2 \sin x = -1$.
 - (b) $\cos^3 x - \cos x = 0$.
 - (c) $4 \sin^2 x + 2(1 - \sqrt{3}) \sin x = \sqrt{3}$.

18. Below are a number of different trigonometric expressions involving or related to $\sin x$. Identify which expressions are equal to each other. (Don't expect them to all pair off!)

- (a) $\sin^2 x$ (b) $\sin(x^2)$ (c) $\csc x$ (d) $(\sin x)^2$ (e) $\sin^{-1} x$
(f) $2 \sin x$ (g) $(\sin x)^{-1}$ (h) $\arcsin x$ (i) $\frac{1}{\sin x}$ (j) $\sin 2x$
(k) $(\csc x^2)^{-1}$ (l) $\frac{\sin 2x}{\cos x}$ (m) $1 - \cos^2 x$ (n) $\cos^2 x - \cos 2x$

19. What are the zeros and asymptotes of

$$\frac{x^2 - 4}{x^4 - 9x^2} ? \quad \frac{x - 4}{x^2 - 3x - 4} ?$$

20. (a) Sketch two functions which both have domain $[0, 20]$ and range $[-10, 10]$ but which do not intersect.

(b) Sketch a function which has domain $[-10, 10)$ and range $[0, \infty)$.

21. What is $\arcsin(\sin x)$? Are you sure about that?

Preparation for next time

Read Section 1.2. Draw the graph of a function $f(x)$ such that

$$\lim_{x \rightarrow 2} f(x) = 4, \quad f(2) = 0.$$