Math 220 AD9 Spring 2009 Worksheet 29

1. Differentiate the following functions:

$$x^3+5, \qquad x^3-7, \qquad x^3+1, \qquad x^3-13.$$

   The function $F(x)$ is an *antiderivative* of $f(x)$ if $F'(x)=f(x)$.
   Find an antiderivative of $3x^2$. Find a different antiderivative of $3x^2$. Describe *all* the antiderivatives of $3x^2$. How do you know that this is all of them? (We discussed this earlier this semester.) Sketch graphs of some antiderivatives of $3x^2$. What is the relation between the graphs of all the antiderivatives of a function $f(x)$? Explain.

2. Find an antiderivative of $8x^3+9x^2+2$. Find another antiderivative of this function. Find all the antiderivatives of this function.

3. We find an antiderivative by "reversing" or "undoing" differentiation. So x^6 and x^6+1 are both antiderivatives of $6x^5$. All of the antiderivatives of $6x^5$ are of the form x^6+c, where c is any constant.

   We call x^6+c the *indefinite integral* of $6x^5$ – it is indefinite because c could be any real number. If $F(x)$ is the antiderivative of $f(x)$, then

$$\int f(x)\,dx = F(x)+c,$$

   is the *indefinite integral* of $f(x)$, where c is an arbitrary constant (called the *constant of integration*).

   Yes, include the dx every time you write down an integral – we will talk more about what the dx means later.

   Always include the constant of integration in your answer.

   What is the indefinite integral of $\sqrt{x}$? What is $\displaystyle\int x^3+2x-7\,dx$?

4. We can see that any two antiderivates of the same function must differ only by a constant. If F_1 and F_2 are both antiderivatives of f, then $F_1'(x)-F_2'(x)=f(x)-f(x)=0$, so F_1 and F_2 have the same derivatives, which shows that they can only differ by a constant. Consider the following functions:

$$F_1(x)=\frac{1}{1-x} \qquad \text{and} \qquad F_2(x)=\frac{x}{1-x}.$$

   Show that these are both antiderivatives of $f(x)=1/(1-x)^2$. Explain why this is not a contradiction.

5. What is the power rule for derivatives? What is the power rule for integration, i.e., what is $\displaystyle\int x^n\,dx$?

   What is $\displaystyle\int 3x^7+x^4-\sqrt[3]{x}+x^{0.1}-3+\frac{1}{x}+\frac{1}{x^3}\,dx$?

6. Joe throws a ball straight upward at an initial speed of $40 ft/s$. Gravitational acceleration is $32 ft/s^2$ downwards. Find a function that describes the position of the ball at each time t.

7. Evaluate the following:

$$\int \sin x\, dx, \qquad \int \cos x\, dx, \qquad \int e^x\, dx, \qquad \int \ln|x|\, dx$$

   Check your answers by differentiating.

8. Find a function $f(x)$ satisfying the following conditions: $f'(x) = \sin x + 1$, $f(\pi/2) = -3$.

9. Find all the functions $f(x)$ satisfying $f'''(x) = \sqrt{x} + 2\cos x$.

10. Evaluate the following:

$$\int x^{1/4}\left(x^{2/3} - x^{-1/2}\right)\, dx, \qquad \int \frac{(e^x)^4 - 5}{e^x}\, dx.$$

11. Determine the position function if the acceleration function is $a(t) = 3\sin x$, the initial velocity is $-2 ft/s$, and the initial position is $s(0) = 4$.

12. Evaluate the following:

$$\int \frac{1}{\sqrt{1-x^2}}\, dx, \qquad \int \frac{1}{1+x^2}\, dx, \qquad \int \frac{x^2+1}{x}\, dx, \qquad \int \frac{1}{\sqrt[3]{2}}\, dx$$

13. What is the derivative of $f(x) = \sin(e^x)$?
    Then what is $\int \cos(e^x)\, e^x dx$? (Don't forget $+C$!!!)

14. What is the derivative of $f(x) = \left(x^3 + x^2 + 1\right)^{10}$?
    Then what is $\int 10\left(x^3 + x^2 + 1\right)^9 \left(3x^2 + 2x\right) dx$? (Don't forget $+C$!!!)

Preparation for next time

For Monday, read section 4.2. There will be a preparation quiz.