Math 220 AD9 Spring 2009 Worksheet 28

1. Suppose that a 6-ft.-tall person is 12 ft away from an 18-ft.-tall lamppost. If the person is walking away at a rate of 2 ft/s, at what rate is the length of the person's shadow changing?

2. A circular oil slick of uniform thickness is caused by a spill of 1,000,000 cm^3 of oil. The thickness of the oil slick is decreasing at a rate of 0.1 cm/hr. At what rate is the radius of the slick increasing when the radius is 8 m?

3. Evaluate the following limits:

$$\lim_{x\to 0}\frac{\ln x^2}{e^{1/x}}, \qquad \lim_{x\to\frac{pi}{2}}\tan x - \sec x, \qquad \lim_{x\to 2}\frac{x^3 - 3x^2 + x + 1}{x - 2}.$$

4. Find and classify the critical points of the following functions:

   (a) $x^3 - 3x$
   (b) $(x + 2)^{2/3}$
   (c) $\frac{x}{x^2 - x - 6}$
   (d) $(1 + x)^{1/3}$

   Classify (a) using both the first derivative test and the second derivative test.

5. Where are the above functions increasing? Decreasing?

6. Where are the above functions concave up? Concave down? What are the points of inflection of the above functions? (Quite messy for (c), but the others can be quickly solved.)

7. Graph the function $x^3 - 3x$.

8. Graph the function $\dfrac{2x^2}{x^2 - 1}$.

9. Find the maximum and minimum values attained by the function $f(x) = x^2 e^{-x^2}$ on the interval $[0, 5]$.

10. What is the largest rectangle whose base in on the x-axis and whose two upper vertices lie on the parabola $y = 4 - x^2$?

11. What is the largest rectangle whose diagonals have length 16?

12. Show that the rectangle of largest area for a given perimeter P is always a square.

13.

Preparation for next time

For Monday, read section 4.1. There will be a preparation quiz.