Math 220 AD9 Spring 2009 Worksheet 27

1. The following steps are useful when solving an optimization problem:
   - Sketch a picture of the situation.
   - Label your sketch, picking out important features.
   - How are the variables related?
   - Write down the (one-variable) function you are trying to optimize?
   - What are the largest and smallest variable values allowed?
   - Solve the problem!
2. Find the point on the curve $y = x^2$ closest to the point $(0, 1)$.
3. An advertisement consists of a rectangular printed region plus 1-in. margines on the sides and 1.5in margins at top and bottom. If the total area of the advertisement is to be $120 in.^2$, what dimensions should the advertisement be to maximize the area of the printed region?
4. You have a piece of wire that is two feet long and you are going to cut it into two pieces, one of which will be bent into a square and one which will be bent into a circle.
   (a) Generate a function which gives the total area of the square the circle.
   (b) Find where you should cut the wire to have the maximum area.
   (c) Find where you should cut the wire to have the minimum area.
5. Related Rates:
   - Sketch the situation.
   - Label the sketch.
   - What do you know? What are you trying to find?
   - What is the relation between the variables/functions?
   - Use implicit differentiation to find the relation between the rates of change.
   - Are you interested in what is happening at some particular point/time? If so, now fill in for that piece of information.
6. Each edge of a square is increasing at a rate of 2 in/s. At what rate is the area of the square increasing when each edge is 10 in?
7. A pebble dropped into a lake creates an expanding circular ripple. Suppose that the radius of the circle is increasing at a rate of 2 in/s. At what rate is its area increasing when the radius is 10 in?

8. A cubical block is melting in such a way that each edge decreases steadily by 2 in every hour. At what rate is its volume decreasing when each edge is 10 in long?
9. A conical water tank has radius 160cm and height 800cm. Water is running out of the tank out of a small hole at the tip (at the bottom). When the height h of the water in the tank is 600cm, what is the rate of change of its volume with respect to h?
10. Falling sand forms a conical sandpile. Its height h always remains twice the radius r while both are increasing. If sand falls at a rate of ft^3/min how fast is r increasing when $r = 5\text{ft}$?
11. A ladder 41 ft tall that was leaning against a vertical wall begins to slip. Its top slides down the wall while its bottom moves along the ground at a constant speed of 4 ft/s. How fast is the top of the ladder moving when it is 9 ft above the ground?
12. A circular oil slick of uniform thickness is caused by a spill of 1,000,000 cm^3 of oil. The thickness of the oil slick is decreasing at a rate of 0.1 cm/hr. At what rate is the radius of the slick increasing when the radius is 8 m?
13. Suppose an ostrich 5 ft tall is walking at a speed of 4 ft/s directly towards a street light 10 ft high. How fast is the tip of the ostrich's shadow moving along the ground? At what rate is the ostrich's shadow decreasing in length?

Preparation for next time

Reminder: You have an exam on Friday – Friday's class will be mainly review.

For Monday, read section 4.1.