

Math 220 AD9 Spring 2009 Worksheet 26

- Here are some useful things to know when sketching a function:
 - Domain
 - Location and type of critical point (local maximum, local minimum, asymptote, “corner”, vertical tangent line, other)
 - Vertical asymptotes
 - Horizontal asymptotes
 - Intervals of increase and decrease
 - Intervals where f is concave up and concave down
 - Points of inflection
 - Global behaviour (behaviour for large x)
 - x - and y -intercepts

How do you find each of the above?

- Sketch the graph of $y = \frac{x^2}{x^2 - 9}$.

- What is a slant asymptote? How do you find a slant asymptote? Graph the function

$$f(x) = \frac{x^3 - 2x^2 + 1}{x^2}$$

and its asymptote on the same axis.

- Find a function which has $x = 1$, $y = 2$, and $x = 3$ as its asymptotes.
- How do you find the global maximum and global minimum of a function on a closed interval?
- You live on a farm and you need to build a (rectangular) pen for your pigs. You are going to build it with one side being your barn, which is 30 meters long. You have 40 meters of fence available. What is the largest pen that you can build for you pigs?
- You have a piece of cardboard that is 10” by 20” and you want to make an open topped box by cutting squares out of the corners and folding up the sides.
 - Sketch this situation.
 - Why do you have to cut out squares?
 - Find the largest box by volume that you can build.
- Find the point on the curve $y = x^2$ closest to the point $(3, 4)$.

9. A three-sided fence is to be built next to a straight section of river, which forms the fourth side of a rectangular region. The enclosed area is to equal $1800ft^2$. Find the minimum perimeter and the dimensions of the corresponding enclosure.
10. An advertisement consists of a rectangular printed region plus 1-in. margins on the sides and 1.5in margins at top and bottom. If the total area of the advertisement is to be $120in.^2$, what dimensions should the advertisement be to maximize the area of the printed region?
11. For any positive constant c , find x to maximize $R(x) = \frac{cx - x^2}{x^2 + c}$.
12. You have a piece of wire that is two feet long and you are going to cut it into two pieces, one of which will be bent into a square and one which will be bent into a circle.
- (a) Generate a function which gives the total area of the square the circle.
 - (b) Find where you should cut the wire to have the maximum area.
 - (c) Find where you should cut the wire to have the minimum area.
13. (For this problem, the important part is the setup and not the tedious calculations needed to get the final answer.)
- You have gone hiking and your car is 5 miles east and 4 miles north of where you are. You are standing in a forest that stretches 3 miles north of you where it borders a field, and the boundary of the field and the forest runs east-west.
- (a) Sketch the forest and field, with your location and the location of your car.
 - (b) If you can hike through the forest at 2 miles per hour and through the field at three miles per hour, generate a function which tells you how long it takes you to reach your car if your path consists of two straight lines, one through the woods and one across the field (it will probably help if you sketch an example).
 - (c) Find the fastest path to your car.
 - (d) Is the shortest path between two points always a straight line?
14. Sketch the graph of $y = x^3 - 3x$.

Preparation for next time

Reminder: You have a quiz on Wednesday and an exam on Friday. There will be a practice exam on Tuesday, 7pm-9pm, in 143 Altgeld Hall. Read sections 3.7 and 3.8 for next time. There will be a preparation quiz.