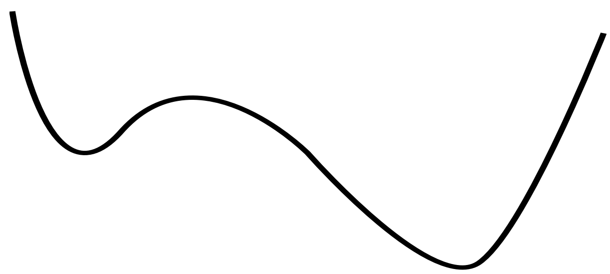
Math 220 AD9 Spring 2009 Worksheet 25

1. Sketch graphs of functions with the following properties:
   (a) $f'(x) > 0$ for all x, $f''(x) > 0$ for $x < -2$ and $x > 2$, $f''(x) < 0$ for $-2 < x < 2$
   (b) $f(1) = 0$, $f'(x) < 0$ for $x < 1$, $f'(x) > 1$ for $x > 1$, $f''(x) < 0$ for $x < 1$ and $x > 1$

2. Look at the following function. Where are its critical points? Where is it concave up and concave down? Where are its points of inflection?

   For each critical point in this graph, explain (using the graph) what $f''(x)$ is at that point. What theorem/test does this illustrate?

   Given a function $f(x)$, how do you find its critical numbers? How do you find its points of inflection? How do you find where it is increasing? How do you find where it is concave down?

3. What are the critical points of $f(x) = x^4 - 18x^2 + 7$? Use the second derivative test to classify these critical points.

   Use the second derivative test to classify the critical points of $g(x) = xe^x$.

4. Where is $x^4 - 18x^2 + 7$ concave up and concave down? Find its points of inflection.

   Where is $g(x) = xe^x$ concave up and down? Find its points of inflection.

5. By now, we know the following facts about $f(x) = x^4 - 18x^2 + 7$:

   - Domain
   - Location and type of critical point
     (local maximum, local minimum, asymptote, "corner", vertical tangent line, other)
   - Vertical asymptotes
   - Horizontal asymptotes
   - Intervals of increase and decrease
   - Intervals where f is concave up and concave down
   - Points of inflection
   - Global behaviour (behaviour for large x)
   - x- and y-intercepts

Use all of this information to sketch the graph of $f(x)$. How about $g(x) = xe^x$?

6. For the function $f(x) = x\ln(x^2)$, find all the points of information listed in the last problem. Use this to sketch the graph of $f(x)$.

7. Use the second derivative test to classify the critical points of $y = x^3$.
   Use the second derivative test to classify the critical points of $y = x^4$.
   Use the second derivative test to classify the critical points of $y = x^{4/3}$.
   Summarise your conclusions.

8. Find the points of inflection of $y = x^4$.
   Find the points of inflection of $y = \sqrt[3]{x}$.
   Find the points of inflection of $y = x^{4/3}$.
   Find the points of inflection of $y = \frac{1}{x}$.
   Summarise your conclusions.

9. Sketch the graph of $y = \frac{x^2}{x^2 - 9}$.

10. Sketch the graph of $y = x^3 - 3x$.

11. The "family of functions" $f(x) = x^4 + cx^2$ behaves differently for different values of the constant c. Determine what differences, if any, there are for c being zero, positive, or negative. What would the graph look like for c zero, positive, or negative?

Preparation for next time

Warning: The week after Spring Break, you will have both a quiz (on Wednesday) and an exam (on Friday). Come prepared to work because there will be a lot to talk about. You should review the basics of sketching graphs and of finding maxima and minima of functions.

Have a great Spring Break!