Math 220 AD9 Spring 2009 Worksheet 22

1. Sketch graphs of functions with the following properties:

   (a) $f(x)$ has a local maximum at $x = 2$ but no absolute maximum.

   (b) $f(x)$ has a local minimum at $x = 2$, a local maximum at $x = 6$ and $f(2) > f(6)$.

   Mark where all the extrema are in these graphs. What do they have in common?

2. Sketch graphs of functions with the following properties:

   (a) $f(x)$ has a local minimum at $x = 2$ but $f'(2) \neq 0$.

   (b) $f(x)$ has critical numbers but no local maxima or minima.

3. Define the following:

   (a) *absolute maximum value*

   (b) *absolute minimum value*

   (c) *local maximum value*

   (d) *local minimum value*

   (e) *critical point*

4. Find the critical numbers of the following functions. If possible, use your knowledge of what the curve looks like to say if these are maxima or minima. (Try to use your calculators as little as possible.)

   (a) $x^2 - 6x + 3$

   (b) $-x^3$

   (c) $x^3 - 3x^2 - 9x + 2$

   (d) $\dfrac{x^2 - 3}{x - 2}$

   (e) $x^2 e^x$

   (f) $\tan^2 x$

5. Sketch graphs of functions on $[-2, 3]$ with the following properties:

   (a) $f(x)$ does not have an absolute maximum on $[-2, 3]$.

   (b) $f(x)$ has an absolute maximum and an absolute minimum on $[-2, 3]$ but these are not at critical numbers.

   Given a function $f(x)$ on $[-2, 3]$, how do we find its maxima and minima? Where can they be? Does this guarantee that there will be maxima or minima at these points?

6. Find the absolute extrema of the function on the given interval:

   (a) $x^4 - 8x^2 + 3$ on $[-3, 1]$,

   (b) $x^{1/3} + 1$ on $[-2, 3]$,

   (c) $\ln(x^2 - 4)$ on $[-1, 2)$,

   (d) $\arctan x^2$ on $[-1, \frac{1}{\sqrt[4]{3}}]$.

7. Sketch the graph of a function that is continuous on $[-2, 3]$. Does it have absolute extrema? What theorem have you just illustrated?

8. Give an example showing that the following statement is false (i.e, not always true):

   Between any two local minima of $f(x)$, there is a local maximum.

9. Find the maximum and minimum values of the function $f(x) = 5x^{2/3} - x^{5/3}$. (Where does the derivative not exist?)

10. Find the maximum and minimum values of the function $f(x) = 3 - |x - 2|$. (Where does the derivative not exist?)

11. Find the smallest value of m that will make the function $g(x) = mx - 1 + 1/x$ greater than or equal to zero for all positive values of x.

12. Let $f(x) = ax^2 + bx + c$ with $a > 0$. Show that if $f(x) \geq 0$ for all real x then $b^2 - 4ac \leq 0$.

13. For what values of a can $f(x) = x^2 + \frac{a}{x}$ have

    (a) a local maximum at $x = 2$;

    (b) an inflection point at $x = 1$;

    (c) a local maximum?

14. Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$.

15. Find the maximum possible area of a rectangle with diagonals of length 16.

16. Suppose you have two numbers whose sum is 20. What is their greatest possible product?

Preparation for next time

Read Section 3.4. You will be asked about the material from Section 3.4 at the beginning of the next merit section.