Math 220 AD9 Spring 2009 Worksheet 22

1. What does it mean to say a limit is of the indeterminate form $\frac{\infty}{\infty}$? Give examples of where a limit of this form is $0, 7$, and ∞. Which of the following are indeterminate forms? For those that are not, give the value of the form.

$$\infty^0 \quad 0^\infty \quad 0^0 \quad \infty - \infty \quad \infty + \infty \quad 1^\infty$$

$$\infty * \infty \quad \infty * (-\infty) \quad 0 * \infty \quad \frac{0}{0} \quad \frac{\infty}{\infty} \quad \infty^\infty$$

2. Consider the following limit:

$$\lim_{x \to 2} \frac{x^3 - 3x - 2}{x^2 + 3x - 10}$$

Near $x = 2$, the numerator can be approximated by $9(x - 2)$ and the denominator can be approximated by $7(x - 2)$ – why? Now that you know this, what would you expect this limit to be? Explain your answer.

In a similar vein, what should we expect the following limits to be?

$$\lim_{x \to \infty} \frac{2x + 3}{5x + 1}, \qquad \lim_{x \to 0} \frac{x}{x^3}, \qquad \lim_{x \to \infty} \frac{x^5 + 2}{x^3 - 17}$$

What does all this have to do with L'Hôpital's rule?

3. To what two indeterminate forms does L'Hôpital's Rule apply? Give an example of a limit with indeterminate form $0 * \infty$ in which applying L'Hopital's Rule directly yields an incorrect answer. What must you always do before applying L'Hôpital's rule?

Important: L'Hôpital's rule only works in these two cases. It will *not* give you the correct answer in any other case.

4. Evaluate the following limits.

$$\lim_{x \to 1} \frac{e^{x-1} - 1}{x^2 - 1}, \quad \lim_{x \to 4} \frac{x^2 - 16x + 4}{x - 4}, \quad \lim_{x \to 0} \frac{\sin x}{e^{3x} - 1}, \quad \lim_{x \to 0} \left(\cot^2 x - \frac{1}{x^2} \right)$$

5. Evaluate the following limits:

$$\lim_{x \to \infty} \frac{\ln x}{x}, \qquad \lim_{x \to 0} \frac{\tan x - x}{x^3}, \qquad \lim_{x \to \pi^-} \frac{\sin x}{1 - \cos x}, \qquad \lim_{x \to 0^+} x \ln x,$$

$$\lim_{x \to \pi/2^-} \sec x - \tan x, \quad \lim_{x \to 0^+} (1 + \sin 4x)^{\cot x}, \qquad \lim_{x \to 0^+} x^x, \qquad \lim_{x \to 0^+} \ln x - x$$

6. Give examples of differentiable functions f and g such that

$$\lim_{x \to \infty} f(x) = \infty = \lim_{x \to \infty} g(x) \qquad \text{and} \qquad \lim_{x \to \infty} f(x) - g(x) = 25.$$

7. We say a function $f(x)$ *dominates* a function $g(x)$ as $x \to \infty$ if $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} g(x) = \infty$ and $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty$.

   What does this mean?

   Do either of these functions dominate the other?

   - e^x or x^5
   - $\ln x$ or x^p (where p is any positive constant)

   Justify your answers.

8. Apply L'Hôpital's Rule. Be sure to convert to the proper form if necessary.

   (a) $\lim_{x\to 0} \dfrac{\sin x}{x}$

   (b) $\lim_{x\to 0} \dfrac{\sin x}{x^2}$

   (c) $\lim_{x\to 0} \dfrac{\sin x}{\cos x}$

   (d) $\lim_{x\to 0} \dfrac{\sin(ax)}{x}$

   (e) $\lim_{x\to 3} \dfrac{4x-2}{(x-3)^2}$

   (f) $\lim_{x\to 3} \dfrac{x^2-9}{x-3}$

   (g) $\lim_{x\to\infty} xe^{-x}$

   (h) $\lim_{x\to-\infty} xe^{-x}$

   (i) $\lim_{x\to\infty} \dfrac{\log(\log x)}{\sqrt{x}}$

   (j) $\lim_{x\to\infty} \dfrac{\log(\log x)}{\log x}$

   (k) $\lim_{x\to 2} \dfrac{1}{x-2} - \dfrac{x+1}{x-2}$

9. What is

$$\lim_{x\to 0} \frac{x}{|x|}?$$

Preparation for next time

Read Section 3.3. You will be asked about the material from Section 3.3 at the beginning of the next merit section.