Math 220 AD9 Spring 2009 Worksheet 20

1. Consider the line $f(x) = 3x + 1$. We have that $f(3) = 10$. Suppose we change the x-value by Δx. What will the change in the function's value be? Write down a formula for $f(x + \Delta x)$.

   Your textbook use Δx to mean the change in the value of x and Δy for the change that then results in the value of y.

2. There is a function $f(x)$ about which you have limited information – for some values of x it is easy to find $f(x)$ but for others it is a real nuisance. You know the following pieces of information:

   $$f(2) = 3, \qquad f(5) = 7, \qquad f'(2) = -1.$$

   Sketch as much of a graph of $f(x)$ as you can.

   What is the equation of the tangent line to $f(x)$ at $x = 2$? Add this tangent line to the sketch from above.

   Can you estimate the value of $f(2.2)$? If so, how? Can you estimate the value of $f(9)$? If so, how?

3. What is the equation of the tangent line to $f(x)$ at $x = x_0$? This is called the *linearization* of or the *linear approximation* to the function $f(x)$ at $x = x_0$.

   What is the linear approximation to $f(x) = x^2 + 1$ at $x = 1$? Draw a graph showing both $f(x)$ and its linearization. Use the linearization to estimate $f(0.9)$, $f(1.1)$, and $f(2)$. Compare these to the actual values.

4. What is $\sin \frac{\pi}{6}$? What is the equation of the tangent line to $y = \sin x$ at the point $x = \frac{\pi}{6}$?

   Note that $\frac{\pi}{6} \approx 0.52$. Estimate the value of $\sin \frac{1}{2}$. How does this compare with the answer your calculator gives you?

   Use the linear approximation to $\sin x$ at $x = \frac{\pi}{6}$ to estimate the value of $\sin 10$. How does this compare with the answer your calculator gives you? Draw a picture that explains where the estimates for $\sin 0.5$ and $\sin 10$ come from.

5. What should the values of $\cos x$ be close to when the values of x are small? Check your guess by finding a linear approximation of $f(x) = \cos x$ about $x = 0$. Do you notice anything peculiar? Why did this happen? (What's special about the point x=0 for cosine?)

6. Use linear approximations to estimate the value of $\sqrt[3]{1001}$. (What function and what center point should you use?)

7. Find the approximate values below by the same means as above.

   (a) $\sqrt[3]{29}$
   (b) $\sqrt{103}$
   (c) $\sqrt[4]{17}$
   (d) $e^{1/10}$
   (e) $(63)^{-2/3}$
   (f) $\sin(28°)$
   (g) $\ln\left(\frac{9}{10}\right)$
   (h) $\sqrt{83}$

8. Consider the expression $y = 3x^2 - \frac{4}{x^2}$. Perform implicit differentiation to find $\frac{dy}{dx}$ Now 'multiply both sides by dx' to find the differential dy.

9. Now do the same for $y = \sin x$ and $y^2 + x^2 = 1$.

10. Use the idea of linear approximation to estimate the change in the given quantities:

    (a) The circumference of a circle if its radius is increased from 10 in to 10.5 in.

    (b) The radius of the earth is approximately 3960 mi around the equator. Suppose that a chain is wrapped tightly around the earth at the equator. Approximately how much does this chain have to be lengthened so that it can be strung around the earth on 10 ft poles?

Preparation for next time

Read Section 3.1 and do problem 21(a).