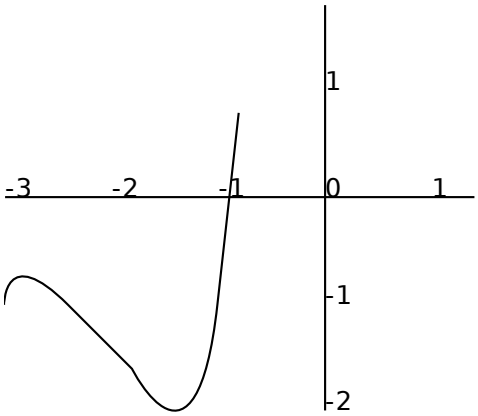
Math 220 AD9 Spring 2009 Worksheet 2

1. A rope is tied snugly around the equator of the earth. 20 metres of extra rope is now added to the old rope. The new rope is now held in a circular shape centred about the earth. Which of the following can now walk underneath the rope without touching it: an amoeba, an ant, or you?

2. The following is a portion of the graph of $f(x)$, which is periodic with period 2. Sketch the graphs of $f(x)$, $f(2x)$, and $f(1-x)$ between 0 and 5.



3. What is a degree? What is a radian? Explain, using pictures. What is the sine of an angle? What is the cosine? Why are these periodic?

4. Sketch the graphs of the following functions:

   (a) $\sin x$

   (b) $5\sin 3x$

   (c) $3\cos\frac{2x}{3}$

   (d) $\tan x$

   (e) $\csc x$

   (f) $\arcsin x$

   (g) $\cos(x+\frac{\pi}{2})$

   Mark the period and amplitude on each graph. What is the domain and range of all the above functions?

5. Which of the above functions are even? odd? Give an example of a polynomial function which is even (odd). How can you tell by looking at a polynomial whether it is even, odd or neither? How can you tell by looking at the graph of a function whether it is even, odd, or neither?

6. Convert the following angles from degrees to radians, then draw the unit circle and label each angle on it in radians.
   (a) $0°$ (b) $30°$ (c) $45°$ (d) $60°$ (e) $90°$
   (f) $120°$ (g) $180°$ (h) $270°$ (i) $360°$ (j) $300°$

7. Using right triangles, the unit circle, or trig identities, calculate the *exact* values $\sin x$ takes at the radian angles that were your answers to the above problem. Do not use your calculator. This course (and all your future math courses) will assume you can find $\sin, \cos, \tan$ of the above angles quickly without a calculator.

8. If $\cos\theta = \frac{3}{5}$ and $0 < \theta < \pi$, find $\sin\theta$ and $\tan\theta$. What if $\pi < \theta < \frac{3\pi}{4}$?

9. Find $\sec(\tan^{-1}(x))$. (Hint:Draw a triangle.)

10. Find all values of θ for which
    (a) $\sin\theta = \tan\theta$
    (b) $\sin 2\theta = \cos\theta$

11. Prove the Pythagorean relationships
    (a) $\sin^2\theta + \cos^2\theta = 1$
    (b) $1 + \tan^2\theta = \sec^2\theta$
    (c) $1 + \cot^2\theta = \csc^2\theta$

12. Without using your calculator, find the exact values of:
    (a) $\ln(e^{\sqrt{2}})$
    (b) $\log_6(\frac{1}{36})$

13. Solve each equation for x:
    (a) $\ln(2x-1) = 3$
    (b) $e^{e^x} = 4$
    (c) $\log_x 32 = 5$
    (d) $\ln(x^2-\pi^2) - \ln(x+\pi) = 7$ (also find the domain of $\ln(x^2-\pi^2) - \ln(x+\pi)$.)

14. (a) Sketch two functions which both have domain [0, 20] and range [-10,10] but which do not intersect.
    (b) Sketch a function which has domain $[-10, 10)$ and range $[0, \infty)$.

15. Without using your calculator, estimate $\log_4 \frac{1}{10}$, $\log_4 60$, $\log_3 7$.

16. With your group compile a list of basic trigonometry facts you should all know.

Preparation for next time

Read Section 0.6. Do problem 2, p.68.