

Math 220 AD9 Spring 2009 Worksheet 19

1. Use the definition of the derivative to find the derivative of $y = x^2 + 2x + 1$.
2. Differentiate the following functions:

$$(a) 7^x \quad (b) e^x \quad (c) x^e \quad (d) \pi^e \quad (e) (x+1)^{(2x^2+1)}$$

3. Differentiate the following functions:

$$(a) (2x+3)\sqrt{x+5} \quad (b) x^2(x^3+1)(2x-1) \quad (c) \frac{x^4 - \sqrt[3]{x+1}}{\ln x} \quad (d) e^{\sin(x^3+1)}$$

4. Consider the functions:

$$\sinh x = \frac{e^x - e^{-x}}{2}, \quad \cosh x = \frac{e^x + e^{-x}}{2}.$$

Show that the derivative of $\sinh x$ is $\cosh x$ and vice versa.

5. Using the Mean Value Theorem, show that for all x and y ,

$$\cos x - \cos y \leq |x - y|.$$

6. Find the equation of the tangent line to the curve $4x^2 + 9y^2 = 25$ at the point $(2, -1)$. Find all the points where the tangent lines are either horizontal or vertical.
7. If f and g are differentiable functions on the interval $[a, b]$ with $f(a) = g(a)$ and $f(b) = g(b)$, prove that at some point in the interval $[a, b]$, f and g have parallel tangent lines.
8. Differentiate the function $f(x) = \arctan \frac{x}{x+1}$ - this could also be written $\tan^{-1} \frac{x}{x+1}$.
9. Find all function $g(x)$ such that $g'(x) = \sqrt{x}$.

10. Find an equation for the tangent line to the curve

$$y^2 + xe^y = 4 - x,$$

at the point $(2, 0)$.

11. The height of a ball at time t is given (in feet) by

$$s(t) = -16t^2 + 64t + 32.$$

At what speed was the ball initially thrown into the air? From where? What is the greatest height it reaches?

12. Find $f^{(34)}(x)$ for $f(x) = \cos 2x$.

Good luck in the exam!