

## Math 220 AD9 Spring 2009 Worksheet 18

1. Find the derivative of  $y(x)$ .

$$(a) \sin(xy) = e^{xy} \quad (b) \ x \arccos(3x^2) \quad (c) \ \frac{x+1}{\arcsin(2x-1)}$$

2. A car is driving along a road with a speed limit of 70 mi/hr. At 3:00 pm the odometer of the car says 10 000 mi. At 4:00 pm the odometer says 10 071 mi. Has this car violated the speed limit during its trip?
3. What does the Mean Value Theorem tells us about the function  $y = x^2$  on the interval  $(-1, 4)$ ? Find the point which satisfies the conclusion of the Mean Value Theorem. (Your answer should include an informative picture.)
4. What does the Mean Value Theorem tell us about the function  $y = \frac{1}{x^2}$  on the interval  $(-2, \frac{1}{2})$ ? Find the point which satisfies the conclusion of the Mean Value Theorem. (Your answer should include an informative picture.)
5. What does the Mean Value Theorem tell us about the function  $y = \frac{1}{x}$  on the interval  $(-1, 1)$ ? Does there exist a point satisfying the conclusion of the Mean Value Theorem? (Your answer should include an informative picture.) Summarize what we learn from this problem and the preceding two problems.
6. Use Rolle's theorem to show that the equation  $x^3 + 2x + c = 0$  cannot have more than one real root for any value of  $c$ .
7. Use the Mean Value Theorem to explain why  $h'(x) = 0$  for all  $x$  implies that  $h$  is a constant function.
8. Find all the functions such that  $f(x) = 3x^2$ . (Hint: Find one function  $f(x)$  such that  $f'(x) = 3x^2$ . If  $g(x)$  is another function with  $g'(x) = 3x^2$ , then ...)
9. Find all the functions  $f(x)$  such that  $f'(x) = \sqrt{x}$ .
10. You are at the greyhound track. You have bet your life savings on Santa's Little Helper, whose main rival is Easter Bunny's Average-Sized Assistant. Their positions at time  $t$  are given by  $s(t)$  and  $b(t)$ . It's a close race but keeping track of things using your radar gun (which you stole after being kicked out of the state police for misusing your radar gun), you see that  $s'(t) > b'(t)$  for all  $t$ . The race announcer declares: "And it's Easter Bunny's Average-Sized Assistant by a nose." Should you lodge a complaint (complete with astute mathematical reasoning) with the racing authorities or should you accept the result, start running, and hope your bookie never catches up with you?
11. Consider the function  $f(x) = x^3 - x$ .
  - (a) What does Rolle's Theorem tell you about  $f(x)$  between  $x = -1$  and  $x = 1$ ?

- (b) Find the  $c$  which satisfies the conclusion of Rolle's Theorem. How many are there?  
How many does Rolle's Theorem say there will be?
- (c) What does Rolle's Theorem tell you about  $f(x)$  between  $x = -2$  and  $x = 1$ ?
12. The function  $f(x) = \frac{(x-1)(x-3)(x+10)}{(x-2)^2}$  has zeroes at 1, 3, and 10. What does the Mean Value Theorem tell us about the location of zeroes of  $f'(x)$ ?
13. Use the Mean Value Theorem to prove Rolle's Theorem.

### **Preparation for next time**

Write a good exam problem based on the following section of Chapter 2: