

Math 220 AD9 Spring 2009 Worksheet 15

1. Draw the graph of $y = \cos x$. Use this to sketch the graph of the derivative of $\cos x$. Use well known trig formulas to write out $\frac{\cos(x+h) - \cos(x)}{h}$. Now take the limit as $h \rightarrow 0$. What is the derivative of $\cos(x)$?
2. You have seen how to find the derivative of $\sin x$ in class. Now use the fact that $\sin(x) = \cos(\frac{\pi}{2} - x)$ to find the derivative of $\cos(x)$.
3. Now use the fact that $\tan(x) = \frac{\sin(x)}{\cos(x)}$ to find the derivative of $\tan(x)$.
4. Now use the fact that $\sec(x) = \frac{1}{\cos(x)}$ to find the derivative of $\sec(x)$.
5. What is the derivative of $\csc x$? $\cot x$? You will be expected to be able to take the derivatives of the six main trigonometric functions very quickly.
6. Find the derivative of $\sin^2 x + \sin x^2 + \sin 2x$.
7. Find the derivatives of the following functions:
 - (a) $f(x) = \frac{\sin x}{x}$
 - (b) $f(x) = \sin 4x \cos^3 x$
 - (c) $f(x) = (1 + \sin(3x))^2$
 - (d) $f(x) = 2x \sin x - 3x^2 \cos x$
 - (e) $f(x) = \sec^2 x - \tan^2 x$
 - (f) $f(x) = \sqrt{\cos \sqrt{x}}$
 - (g) $f(x) = \sqrt{x} \sec \sqrt{x}$
 - (h) $f(x) = \frac{\sin x}{\sec x}$
8. Where is the line tangent to $y = \sin x \cos x$ horizontal?
9. Find the derivative of $f(x) = \sqrt{\sin x} - \sin \sqrt{x}$.
10. Find the derivative of $f(x) = \sec^2 x - \tan^2 x$.
11. Find the derivatives of the following functions:
 - (a) $\sqrt{\cos \sqrt{x}}$
 - (b) $\frac{\cos x}{x}$
 - (c) $\sin^3 x^2$
12. Differentiate the double angle identity $\cos 2x = \cos^2 x - \sin^2 x$ to obtain the double angle identity for sine. If you take another derivative, do you get back the first formula?
13. Differentiate the addition formula $\sin(x+b) = \sin x \cos b + \cos x \sin b$ to obtain the addition formula for cosine.
14. Suppose $y = \arcsin x$. Then $x = \dots$? Differentiate both sides (using the chain rule). Now find the derivative of $\arcsin x$.
Repeat for $\arccos x$ and $\arctan x$.

15. The range (total horizontal distance traveled) of a projectile fired from the ground with initial speed v_0 at an angle of θ is given by $R = \frac{v_0^2}{16} \sin \theta \cos \theta$. Find the angle that maximizes the range.

16. The amount of sunlight per day in Chicago can be approximated by the function

$$t(x) = 12 + (2.8833) \sin \left(\frac{2\pi x}{365} \right).$$

(a) What point on the sine curve are we on right now?

(b) How much more sunlight was there yesterday than today?

(c) How quickly is the amount of sunlight decreasing right now?

17. Suppose you know that $f(1) = 1$ and $f'(1) = 3$. Find the derivative of the function $g(x) = \frac{f(x^2)}{x^3}$.

18. The biggest challenge in computing the derivatives of $\sqrt{(x^2 + 4)(x^3 - x + 1)}$, $(x^2 + 4)\sqrt{x^3 - x + 1}$, and $x^2 + 4\sqrt{x^3 - x + 1}$ is knowing which rule (product, chain, ...) to use when. Discuss how you know which rule to use when.

Preparation for next time

Read section 2.7. Do problem 2, p. 214.