

Math 220 AD9 Spring 2009 Worksheet 14

- Find the equation of the tangent line to $y = 2x^2 + 3$ at $x = -2$. Explain how to find the equation of the tangent line to $y = f(x)$ at $x = a$.
- Suppose that f and g are differentiable functions. Suppose that the tangent line to $f(x)$ at $x = 0$ is $y = -x - 1$ and the tangent line at $x = 1$ is $y = 3x + 2$. Suppose that the tangent line to $g(x)$ at $x = 0$ is $y = -x - 3$ and the tangent line at $x = 1$ is $y = -2x - 1$. Fill in the following table:

$f(0) =$	$f(1) =$	$g(0) =$	$g(1) =$
$f'(0) =$	$f'(1) =$	$g'(0) =$	$g'(1) =$

Find the equation of the tangent line to $h(x) = f(x)g(x)$ at $x = 1$. Find the equation of the tangent line to $k(x) = \frac{f(x)}{g(x)}$ at $x = 0$.

- Snickers runs 3 times faster than Allie. Allie runs 5 times faster than Min. How many times faster is Snickers than Min? Now explain this using suitable functions and the chain rule.
- Find the derivatives of the following functions:

$$(a) \quad (x^3 + 3x^2 + 4x)^5 \quad (b) \quad \sqrt[3]{5 + x - x^2} \quad (c) \quad \frac{1 + x}{\sqrt{4x - x^3}} \quad (d) \quad \sqrt{\frac{1 + x}{4x - x^3}}$$

- Hugh's watch is broken. It gave the correct time at midnight but is running two hours late by the beginning of class. (Use military time: 0000=midnight, etc..)
 - Devise a function $g(t)$ (based on the time t that Hugh reads on his watch) which gives the real time.
 - Hugh drives to Illinois from New Jersey. A function which gives a good approximation of his position is $f(t) = 70t$. How fast did he drive?
 - Give a function which records the position of the car based on the time that Hugh reads on his watch.
 - In the time that it took to drive 70 miles, how much time elapsed on his watch?
 - What was the velocity of the car according to readings Hugh took from his watch?
- Repeat the previous problem, but with a few changes.
 - The position of the car is now given by the function $f(t) = t^2 + 36t$. How fast is Hugh going at time t ?
 - Hugh's watch is now even more severely damaged, so the real time is given by $g(t) = t^2 + 1/t$. What function gives the position of the car based on the time on his watch?

- (c) What is the rate of change between the real time and the time on his watch at time t ?
- (d) When his watch tells him that it is 2, how fast is the car actually going?
- (e) When his watch tells him that it is 2, how fast is the car going according to my watch?
- (f) When his watch tells him that it is time a , how fast is the car going according to his watch?
- (g) Discuss all of the above with regards to the chain rule.
7. Suppose you know that $f(1) = 1$ and $f'(1) = 3$. Find the derivative of the function $g(x) = \frac{f(x^2)}{x^3}$.
8. Use the product and chain rules to prove the quotient rule.
9. Suppose that g and f are inverse functions, so $f(g(x)) = x$. Differentiate both sides and solve for $g'(x)$.
If $f(x) = \sqrt{x^5 + 4x^3 + 3x + 1}$, and g is its inverse function, find $g'(3)$.
10. The biggest challenge in computing the derivatives of $\sqrt{(x^2 + 4)(x^3 - x + 1)}$, $(x^2 + 4)\sqrt{x^3 - x + 1}$, and $x^2 + 4\sqrt{x^3 - x + 1}$ is knowing which rule (product, chain, ...) to use when. Discuss how you know which rule to use when.
11. Suppose that $g(0) = 0$ and that $g'(0) = 1$. What is the derivative of $g(g(g(x)))$ at $x = 0$?
12. Given that $f'(2) = 2$, $f'(3) = -1$, $g(1) = 2$, $g'(1) = -2$, $g(0) = 3$ and $g'(0) = 8$, calculate $h'(1)$ and $h'(0)$ where $h(x) = f(g(x))$.
13. If $f(x)$ is a differentiable function, find an expression for the derivative of $\sqrt{4f(x) + 1}$.
14. Problems 51-54, p. 195.
15. (a) You are asked to calculate the value of $F'(7)$ for $F = f \circ g$. The functions f and g are unknown, but you are permitted to ask exactly three questions regarding numerical values of these functions and/or their derivatives at specified points. What three questions should you ask?
(b) What if, in the above, $F = f \circ g \circ h$, a composition of three functions... what is the least number of questions you need to ask to calculate $F'(7)$ and what are those questions?
16. Find the first, second, and third derivatives of $f(x) = \frac{2}{x+3}$ and develop a formula for the n th derivative of f .

Preparation for next time

Read section 2.6. Do problem 3, p.203.