Math 220 AD9 Spring 2009 Worksheet 13

Reminder: We know the following rules for differentiation:

$$\frac{d}{dx}k = 0, \text{for any constant } k, \qquad \frac{d}{dx}cf(x) = cf'(x), \text{for any constant } c,$$
$$\frac{d}{dx}\left(f(x) + g(x)\right) = f'(x) + g'(x), \qquad \frac{d}{dx}x^r = rx^{r-1}, \text{for any constant } r.$$

Also, *never* write $x^2 = 2x$ when you mean $\frac{d}{dx}x^2 = 2x$.

1. Suppose that you are changing the size of a image on your computer by dragging the top right corner of the image. Say that the bottom left corner of the image is at the origin.

   (a) If the position of the top right corner of the image is given by the functions $x = f(t) = 5 + 2t$ and $y = g(t) = 7 + 3t$, what is the initial size of the image?

   (b) What is the area of the image at time t?

   (c) How fast is the width of the image changing at time t?

   (d) How fast is the height of the image changing at time t?

   (e) Draw a picture demonstrating the size of the picture at $t = 1$ and $t = 2$. Find the difference in the two areas.

   (f) Draw a picture demonstrating the size of the picture at t and $t + h$. Find the difference in the two areas.

   (g) Divide the equation from the previous problem by h and take the limit as h goes to 0. What do you get?

   (h) Compare this to the product rule for derivatives. Does it look familiar?

2. We know that the derivative with respect to x of $[f(x) + g(x)]$ is $f'(x) + g'(x)$. This might lead you to assume that $\frac{d}{dx}[f(x)g(x)] = f'(x)g'(x) \quad (\dagger)$ for products, but this is **NOT TRUE!!** The *product rule* is

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + g'(x)f(x).$$

   (a) What is the derivative of $y = x^2 = x \cdot x$? What would the derivative be according to $(\dagger)$? Now calculate the derivative using the correct form of the product rule.

   (b) Differentiate $x^3\sqrt{x}$ - calculate this derivative two ways.

   (c) Differentiate $(x^{10} - 3x^7 + x^3 - 3)(2x^{17} + 3x^{11} - 8x^5 + 4x^2)$.

   (d) Differentiate $(x^2 + 5x - 3)^2$

3. Suppose $f(x)$, $g(x)$ and $h(x)$ are all differentiable functions. Write down a product rule for
$$\frac{d}{dx}\Big(f(x)g(x)h(x)\Big).$$
(Hint: use the product rule twice.)

Differentiate $x(2x-1)(3x+1)$. Differentiate $(1+x+x^2)(3+x+x^4)(5+6x+9x^3)$.

4. What is the derivative of $\dfrac{f(x)}{g(x)}$?

Differentiate $x^3 = \dfrac{x^5}{x^2}$ using the quotient rule. Use this example to show that the quotient of the derivatives is *not* the derivative of the quotient.

Differentiate the following functions:

(a) $\dfrac{x^2}{2+3x}$ (b) $\dfrac{7}{x^3+2x+4}$ (c) $\dfrac{1+x+3x^6}{2+x+x^2}$ (d) $\dfrac{x^3-1}{x-1}$ (e) $\dfrac{(x^2+x+1)(2x^3-4x+5)}{2+x^2}$.

5. Suppose that f and g are differentiable functions. Suppose that the tangent line to $f(x)$ at $x=0$ is $y=-x-1$ and the tangent line at $x=1$ is $y=3x+2$. Suppose that the tangent line to $g(x)$ at $x=0$ is $y=-x-3$ and the tangent line at $x=1$ is $y=-2x-1$. Fill in the following table:

$f(0)=$	$f(1)=$	$g(0)=$	$g(1)=$
$f'(0)=$	$f'(1)=$	$g'(0)=$	$g'(1)=$

Find the equation of the tangent line to $h(x)=f(x)g(x)$ at (a) $x=0$, (b) $x=1$.

Find the equation of the tangent line to $k(x)=\dfrac{f(x)}{g(x)}$ at (c) $x=0$, (d) $x=1$.

6. Find the derivatives of the following functions:

(a) $f(x)=(x-1)(x^4+x^3+x^2+x+1)$

(b) $f(x)=x(2x+2)(2x-2)$

(c) $f(x)=(x-1)^3$

(d) $f(x)=\dfrac{x^5-1}{x-1}$

(e) $f(x)=\dfrac{2}{2x^2+x+4}$

(f) $f(x)=\dfrac{\frac{1}{x}-\frac{2}{x^2}}{\frac{2}{x^3}-\frac{3}{x^4}}$

(g) $f(x)=\dfrac{x}{(x-1)^2}$

(h) $f(x)=\dfrac{x+1}{x-1}$

7. Show that the tangent to any point (a,a^3) on the curve $y=x^3$ meets the curve again at a point where the slope is four times the slope at (a,a^3).

Preparation for next time

Read section 2.5. Do problem 1, p.194.

Winner of the office hour plebiscite by a landslide: Tuesdays at 3pm.

Have a good weekend!