Math 220 AD9 Spring 2009 Worksheet 12

Reminder: We know the following rules for differentiation:

$$\frac{d}{dx}k = 0, \text{for any constant } k, \qquad \frac{d}{dx}cf(x) = cf'(x), \text{for any constant } c,$$
$$\frac{d}{dx}\left(f(x) + g(x)\right) = f'(x) + g'(x), \qquad \frac{d}{dx}x^r = rx^{r-1}, \text{for any constant } r.$$

Also, *never* write $x^2 = 2x$ when you mean $\frac{d}{dx}x^2 = 2x$.

1. Let $f(x) = x^5 - x^3 + 7x^2 - 2x + 5$. Find the following:
   (a) $f''(x)$ (b) $f^{(4)}(2)$ (c) $f'''(-1)$ (d) $f^{(6)}(2)$.
   If $g(x) = 10x^4 - 9x^3 + 8x^2 - 7$, find $g^{(17)}(x)$.

2. Draw the graph of a function $f(x)$ for which $f''(x) > 0$ for all x. Draw the graph of a function $g(x)$ for which $g''(x) > 0$ if $x > 0$ and $g''(x) < 0$ if $x < 0$. Does the second derivative tell us anything about the graph? (We will meet this again later.)

3. If $s(t)$ gives the position of a particle at time t, what is the meaning of $s'(t)$, $s''(t)$? What does it mean when $s'(t) < 0$? $s''(t) < 0$? What would be the case if $s''(t) = 0$ for all t?
   If $s(t) = 5 + 2t - t^2$, where is the particle's starting point? When is the particle furthest to the left? The right? When is it accelerating? Decelerating? When does it stop moving (even if only for an instant)?

4. Assume that a is a real number, $f(x)$ is differentiable for all $x \geq a$ and $g(x) = \max_{a \leq t \leq x} f(t)$ for $x \geq a$. Find $g'(x)$ in the cases
   (a) $f'(x) > 0$ and (b) $f'(x) < 0$.

5. Gravity Problems

   (a) Assume that the acceleration due to gravity $g \approx 32\text{ft/s}^2$. Derive the equation for the velocity of the falling object as a function of time. Use v_0 to denote the velocity at time $t = 0$.

   (b) Now use the answer above to calculate the position as a function of time. Use y_0 to be the initial ($t = 0$) position.

   (c) If I told you that a ball thrown upward from a height of zero reached a height of 100ft, what was its initial velocity?

   (d) If a ball is thrown upward with a velocity of 320ft/s, what is its velocity when it reaches the same height on the way back down?

   (e) If a ball is thrown upwards at an initial height of 50ft with a velocity of 20ft/s what will its speed be at a height of 0ft?

6. Does the power rule apply to $f(x) = x^0$? Why or why not?

7. Find a cubic polynomial $f(x)$ such that $f(0) = 0$, $f'(0) = 1$, $f''(0) = 2$, and $f'''(0) = 3$.

8. Find an nth degree polynomial $f(x)$ such that $f^{(k)}(0) = k$ for each $0 \leq k \leq n$.

9. Show that the tangent to any point (a, a^3) on the curve $y = x^3$ meets the curve again at a point where the slope is four times the slope at (a, a^3).

10. We know that the derivative with respect to x of $[f(x) + g(x)]$ is $f'(x) + g'(x)$. This might lead you to assume that $\frac{d}{dx}[f(x)g(x)] = f'(x)g'(x) \quad (\dagger)$ for products, but this is **NOT TRUE!!** The *product rule* is

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + g'(x)f(x).$$

   (a) What is the derivative of $y = x^2 = x \cdot x$? What would the derivative be according to $(\dagger)$? Now calculate the derivative using the correct form of the product rule.

   (b) Differentiate $x^3\sqrt{x}$ - calculate this derivative two ways.

   (c) Differentiate $(x^{10} - 3x^7 + x^3 - 3)(2x^{17} + 3x^{11} - 8x^5 + 4x^2)$.

   (d) Differentiate $(x^2 + 5x - 3)^2$

11. Suppose $f(x)$, $g(x)$ and $h(x)$ are all differentiable functions. Write down a product rule for

$$\frac{d}{dx}\Big(f(x)g(x)h(x)\Big).$$

Differentiate $x(2x - 1)(3x + 1)$. Differentiate $(1 + x + x^2)(3 + x + x^4)(5 + 6x + 9x^3)$.

12. What is the derivative of $\dfrac{f(x)}{g(x)}$?

Differentiate $x^3 = \dfrac{x^5}{x^2}$ using the quotient rule. Use this example to show that the quotient of the derivatives is *not* the derivative of the quotient.
Differentiate the following functions:
(a) $\dfrac{x^2}{2+3x}$ (b) $\dfrac{7}{x^3+2x+4}$ (c) $\dfrac{1+x+3x^6}{2+x+x^2}$ (d) $\dfrac{x^3-1}{x-1}$ (e) $\dfrac{(x^2+x+1)(2x^3-4x+5)}{2+x^2}$.

Preparation for next time

Read Section 2.4. Make a list featuring every differentiation rule you have met so far in this course. Also do Problem 5, p. 186.