

Math 220 AD9 Spring 2009 Worksheet 11

1. Graph $y = 6$. If $y = 6$, what is $\frac{dy}{dx}$? Explain.

Graph $y = -4x$. If $y = -4x$, what is $\frac{dy}{dx}$? Explain.

Graph $y = x^2$. Use this to sketch a plausible graph of $\frac{dy}{dx}$. What is $\frac{dy}{dx}$ in this case?

Repeat for $y = x^3, y = x^4, y = \frac{1}{x}, y = \frac{1}{x^2}$.

Now state the power rule for derivatives and check that it matches up with your work above.

2. Differentiate the following functions:

(a) $x^\pi + e$

(b) $\sqrt[7]{x^9}$

(c) $\frac{5}{x^{11}}$

(d) $-7x^5 + 4x^3 - 9x^2 + x^{\sqrt{2}} + x + 6 + x^{-1}\frac{9}{x^3} + \frac{4}{\sqrt[4]{x^3}}$

3. For each of the following functions, draw a rough sketch of the graph of that function. Using no other information besides that graph, sketch the graphs of the derivatives of the functions below.

$$e^x, \quad \ln x, \quad \sin x.$$

4. (a) Say you have two functions $f(x)$ and $g(x)$, and you know that their derivatives $f'(x)$ and $g'(x)$ exist for all x . Using the definition of derivative, prove that if $h(x) = f(x) + g(x)$ then $h'(x) = f'(x) + g'(x)$.
- (b) Say you have two functions $f(x)$ and $g(x)$, $f(3) = 4, g(3) = -1, f'(3) = \frac{1}{2}, g'(3) = \frac{2}{3}$, find the equation of the tangent line to $[f(x) + g(x)]$ at $x = 3$.
5. Find the parabola $y = ax^2 + bx$ whose tangent line at $(1, 1)$ has equation $y = 3x - 2$.
6. Show that the derivative of an even function is an odd function and that the derivative of an odd function is an even function.
7. Find the derivative of $f(x) = ax^3 + bx^2 + cx + d$, and use it to answer the following questions:
- (a) Prove that the rate of change of the volume of a cube with respect to its edge length is equal to half the surface area.
- (b) Show that the rate of change of the volume of a sphere with respect to its radius is equal to its surface area.
8. Find two points on the curve $y = x^4 - 2x^2 - x$ that have a common tangent line.

9. Find points P and Q on the parabola $y = 1 - x^2$ so that the triangle ABC formed by the x-axis and the tangent lines at P and Q is an equilateral triangle.
10. Show that the tangent to any point (a, a^3) on the curve $y = x^3$ meets the curve again at a point where the slope is four times the slope at (a, a^3) .
11. Find a cubic polynomial $f(x)$ such that $f(0) = 0$, $f'(0) = 1$, $f''(0) = 2$, and $f'''(0)$.
12. Find an n th degree polynomial $f(x)$ such that $f^{(k)}(0) = k$ for each $0 \leq k \leq n$.
13. Gravity Problems
 - (a) Suppose that a scientist tells you that the force of gravity between two objects is given by $F = \frac{GmM}{d^2}$ where m and M are the masses of the two objects, d is the distance between the centers of the two objects, and G is some constant. Now suppose that she tells you that force and acceleration are related by the equation $F = ma$. Let m be the mass of a falling object, and let M be the mass of the earth. Figure out what the falling object's acceleration will be.
 - (b) Now use your answer in the previous question to find acceleration as a function of time (be careful whether you make it positive or negative). Assume that the distance between the object and the center of the earth is constant (on the cosmic scale, this is very close to being true). Now use the abbreviation $g = \frac{GM}{d^2}$ to simplify your equation.
 - (c) Assume that $g \approx 32\text{ft/s}^2$. Derive the equation for the velocity of the falling object as a function of time. Use v_0 to denote the velocity at time, $t = 0$.
 - (d) Now use the answer above to calculate the position as a function of time. Use y_0 to be the initial ($t = 0$) position.
 - (e) If I told you that a ball thrown upward from a height of zero reached a height of 100ft, what was its initial velocity?
 - (f) If a ball is thrown upward with a velocity of 320ft/s, what is its velocity when it reaches the same height on the way back down?
 - (g) If a ball is thrown upwards at an initial height of 50ft with a velocity of 20ft/s what will its speed be at a height of 0ft.
 - (h) Suppose you are an astronaut exploring another planet. If you drop a ball from a height of 64ft and it takes 6 seconds to reach the earth, then what is the acceleration due to gravity on that planet? *bonus* What must the volume of this planet be compared to earth if they have the same mass?

Preparation for next time

Read Section 2.4. Do Problem 1, p. 186.