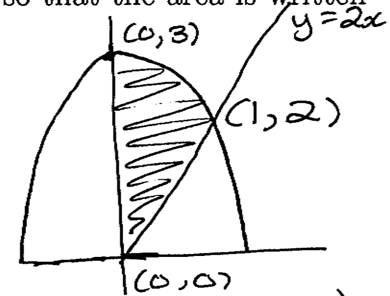
Math 220 AD9, Spring 2009, Quiz 10

Name: *Solutions*

1. (Q. 20, p. 438) Find the area of the region bounded by the curves $y = 2x$ ($x > 0$), $y = 3 - x^2$, and $x = 0$. Choose the variable of integration so that the area is written as a single integral.



Intersect at $(1,2)$

$\left(2x = 3 - x^2 \Rightarrow x^2 + 2x - 3 = 0 \Rightarrow (x+3)(x-1) = 0 \Rightarrow x = -3 \text{ or } x = 1 \right)$

$$\int_0^1 3 - x^2 - 2x \; dx$$

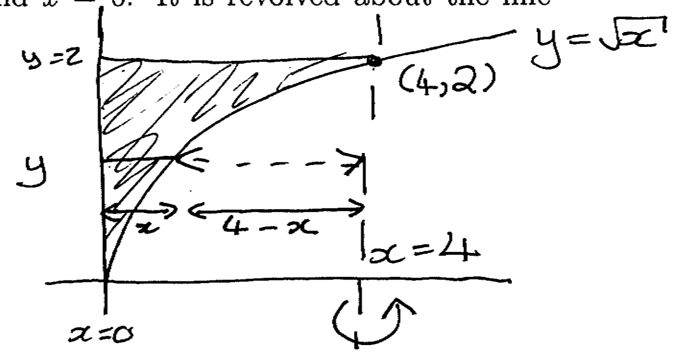
"Top function" − "Bottom function"

(Interested in x between 0 and 1)

$$= 3x - \tfrac{1}{3}x^3 - x^2 \Big|_0^1 = 3 - \tfrac{1}{3} - 1 = \tfrac{5}{3}$$

2. (Q. 19, p. 454) Compute the volume of the solid formed by revolving the given region about the given line.

The region is bounded by $y = \sqrt{x}$, $y = 2$ and $x = 0$. It is revolved about the line $x = 4$.



For each y between 0 and 2 we get a "washer" with inner radius $4 - x$ and outer radius 4.

But our integral should only feature y. How do we write x as a function of y? $y = \sqrt{x}$ so $y^2 = x \Rightarrow$ Inner Radius $= 4 - y^2$

$$\int_0^2 \pi(4^2) - \pi(4 - y^2)^2 \, dy$$

$$= \int_0^2 16\pi - \pi(16 + y^4 - 8y^2) \, dy$$

$$= \pi \int_0^2 8y^2 - y^4 \, dy = \pi\left(\tfrac{8}{3}y^3 - \tfrac{1}{5}y^5\right)\Big|_0^2 = \pi\left(\tfrac{64}{3} - \tfrac{32}{5}\right).$$