Merit Math 220, Spring 2009, Review Sheet for Final Exam

The usual warnings apply. I am not writing the final exam.

1. Evaluate the following limits. Justify your work.

   (a) $\lim_{x \to 0} \dfrac{5x}{\sin 3x}$

   (b) $\lim_{x \to \pi} \dfrac{\cos x}{2x^2 + 3}$

   (c) $\lim_{x \to \infty} \dfrac{2x^3 + 5x^2 + x + 1}{1 - x^2}$

   (d) $\lim_{x \to \infty} \dfrac{\ln(x^2 + 3)}{x}$

2. Consider the following function

$$f(x) = \begin{cases} x^2 + \frac{3}{x} & x \leq 1 \\ 3 & x = 1 \\ 3x + 1 & x > 1 \end{cases}$$

What is $\lim_{x \to 1} f(x)$?

Where is $f(x)$ continuous? Justify your answers.

3. Use the *definition* of the derivative to find the derivative of $f(x) = 3x^2 + 2x - 1$.

4. What does the Mean Value Theorem tell us about $y = x^3 + 1$ on the interval $[0, 2]$?

5. Differentiate the following functions.

(a) $\dfrac{\ln(x^2+3)}{x}$

(b) $(x^2+1)\sin 4x$

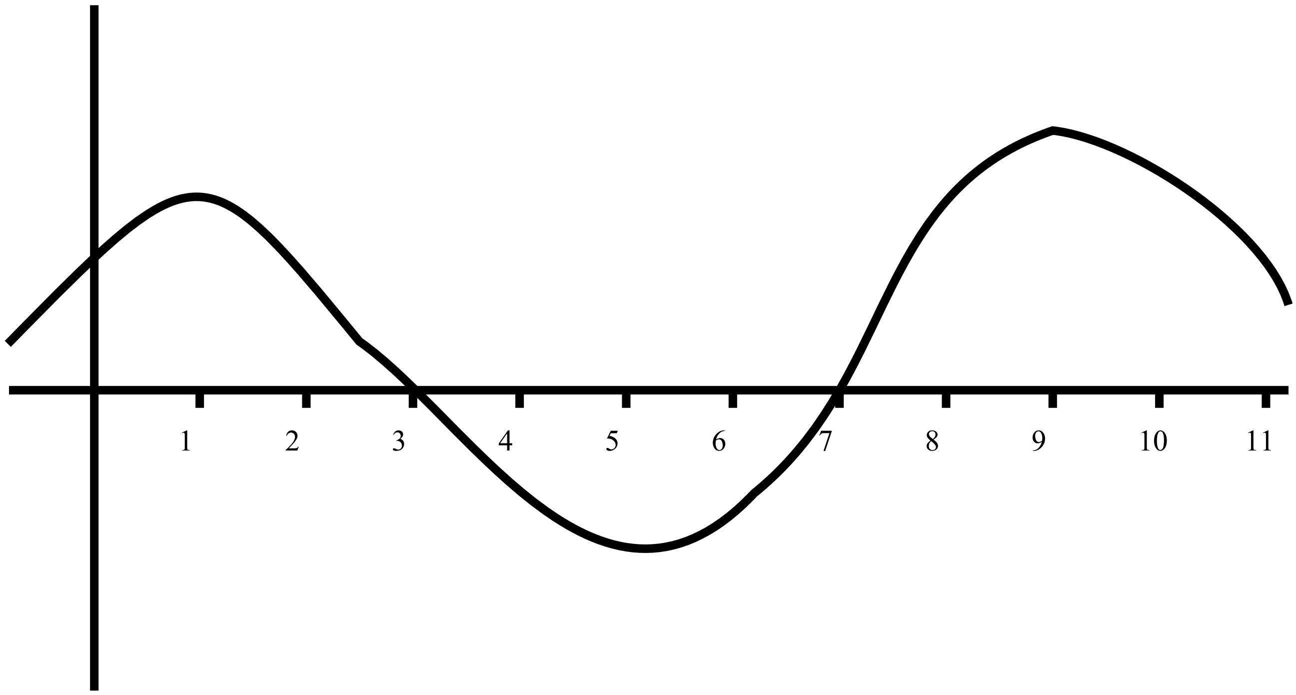
(c) $5x+7+e^{\sqrt{5x+7}}$

6. Find the equation of the tangent line to the curve $x^2 + y^2 = \frac{5}{2}xy$ at the point $(1, 2)$.

7. Use linear approximation to estimate $\sqrt{26}$.

8. Find the maximum and minimum values attained by $f(x) = (x^2 - 1)^{2/3}$ on the interval $[-1, 3]$.

9. The following graph is of the function $y = f(x)$. Define $F(x) = \int_0^x f(x)\,dx$.



(a) Where is the function $F(x)$ increasing? Decreasing?

(b) Where is the function $F(x)$ concave up? Concave down?

(c) Where are the inflection points of $F(x)$?

(d) Classify the critical points of $F(x)$?

10. A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. She needs no fence along the river. What are the dimensions of the field that has the largest area?

11. Find and classify the critical points of the function

$$y = \frac{x^2}{x+3}$$

12. Evaluate the following integrals.

(a) $\displaystyle\int \frac{1}{x(\ln x)^2}\ dx$

(b) $\displaystyle\int x^3 + \frac{1}{x} + \frac{1}{\sqrt{1-x^2}} + \cos 4x\ dx$

(c) $\displaystyle\int_0^1 5xe^{x^2}\ dx$

13. Use a left endpoint Riemann sum with $n = 4$ to estimate

$$\int_{-2}^{2} x^3 dx.$$

14. Suppose that a cylindrical block of ice is melting at the rate of 1 cm^3/s, in such a way that the radius is always twice the height. At what rate is the radius changing when the height is 10 cm?

15. Find the area bounded between the two curves $y = 4 - x^2$ and $y = 3x$

16. Find the volume of the solid obtained by revolving the region from the last problem around the line $y = -12$.

    (At least write down the integral – it gets a little messy after that point.)