Math 242, Merit Worksheet 29, Fall 2005

1. Consider the integral \( \int_{-1}^{1} \int_{x^2}^{1} \int_{0}^{1-y} \, dz \, dy \, dx \).

   (a) What does this integral evaluate for us? (In words)
   (b) Sketch the region of integration.
   (c) Rewrite the integral in the following orders:
   \( dy \, dz \, dx, \, dx \, dy \, dz, \, dz \, dx \, dy \)
   (d) Evaluate the integral.

2. Set up integrals to find the volume of the wedge cut from the cylinder \( x^2 + 9y^2 = 4 \) by the planes \( z = 0 \) and \( z = 3x \). Rewrite the integral in a different order of integration.

3. Set up integrals in rectangular, cylindrical, and spherical coordinates to find the volume of the solid bounded below by the cone \( z^2 = x^2 + y^2 \) and bounded above by the plane \( z = 3 \). Which coordinate system seems most promising here?

4. Let \( E \) be the solid region bounded by the lower half-cone \( z = -2\sqrt{x^2 + y^2} \) and the two spheres \( x^2 + y^2 + z^2 = 1, \, x^2 + y^2 + z^2 = 4 \). Express the volume of \( E \) as a triple integral in spherical coordinates.

5. Let \( F \) be the solid region bounded below by the \( xy \)-plane, bounded above by the paraboloid \( z = 9 - x^2 - y^2 \), and on the sides by the cylinder \( x^2 + y^2 - 2x = 0 \). Express the volume of \( F \) as a triple integral in cylindrical coordinates.

If you have any questions, email me to make an appointment. Also, you will probably find me if you check B1 Coble Hall and 150 Altgeld Hall.

Best of Luck in the Final Exam

Happy Holidays!