Math 242, Merit Worksheet 26, Fall 2005

1. Use polar coordinates to combine the sum

\[
\int_{1/\sqrt{3}}^{1} \int_{\sqrt{1-x^2}}^{x} xy \, dy \, dx + \int_{1/\sqrt{2}}^{2} \int_{0}^{\sqrt{2}} xy \, dy \, dx + \int_{\sqrt{1} - \sqrt{2}}^{\sqrt{2}} \int_{0}^{\sqrt{4-x^2}} xy \, dy \, dx
\]

into one double integral and then evaluate.

2. Use a double integral to evaluate the area enclosed by one leaf of the four-leaved rose \( r = \cos 2\theta \).

3. Find the centroid of the rectangular lamina \( \{ -1 \leq x \leq 1, -1 \leq y \leq 1 \} \) with density \( \delta(x, y) = 2y + x^2 \). Can you exploit symmetry in any way?

4. Find the mass and centroid of the region bounded by \( y = 0, x = -1, x = 1 \), and \( y = e^{-x^2} \), with \( \delta(x, y) = |xy| \).

5. Use the first theorem of Pappus to find the centroid of the first-quadrant portion of the annular ring with boundary circles \( x^2 + y^2 = a^2 \) and \( x^2 + y^2 = b^2 \) (where \( 0 < a < b \)).

6. Apply the second theorem of Pappus to find the centroid of the arc that consists of the first-quadrant portion of the circle \( x^2 + y^2 = r^2 \).

7. (a) Apply the first theorem of Pappus to show that the volume of the conical frustum obtained by revolving the trapezoid pictured below around the \( y-axis \) is \( V = (1/3)\pi h(r_1^2 + r_1 r_2 + r_2^2) \).

(b) Apply the second theorem of Pappus to show that the lateral surface area of the conical frustum is \( \pi(r_1 + r_2)L \), where \( L = \sqrt{(r_1 - r_2)^2 + h^2} \).
Warm-up for Thursday

Compute the value of the triple integral

$$\int_0^2 \int_0^3 \int_0^1 x + y + z \, dz \, dy \, dx.$$