Math 242, Merit Worksheet 22, Fall 2005

1. Find and classify the critical points for the function 
   \( f(x, y) = (x^2 + y^2)e^{x^2-y^2} \).

2. Show that the sphere \( x^2 + y^2 + z^2 = r^2 \) and the cone \( z^2 = a^2x^2 + b^2y^2 \) are orthogonal (that is, have perpendicular tangent planes) at every point of their intersection.

3. A wire 120 cm long is cut into three pieces. Each piece is then bent into the shape of a square. Let the function \( f \) be the sum of the area of these three squares. Show that the single critical point of \( f \) is a local minimum. However, it should also be possible to maximise the sum of the areas. Explain.

4. What possible positions could a tangent plane have if \( f_x = 0 \) at the point of tangency?

5. Sketch the regions of integration and evaluate the following integrals by changing the order of integration:

   \( \int_0^1 \int_{y^{1/3}}^1 \frac{1}{\sqrt{1+x^2}} \, dx \, dy \)

   \( \int_0^8 \int_{x^{2/3}}^4 x \cos y^4 \, dy \, dx \)

   \( \int_0^4 \int_{\sqrt{y}}^2 \frac{ye^{x^2}}{x^3} \, dx \, dy \)

6. Find the area in the \( xy \)-plane between the curves \( y = x \) and \( y = 2x^2 - x \). Think about this both as a double integral and a single integral and why they are really the same thing.

7. Find \( \int_D 3 \, dA \) where \( D \) is the circular region in the \( xy \)-plane with centre \( C(10, -50) \) and radius 2. What if the centre is at \( C(-1, 15) \)?

Warm-Up for Thursday

Evaluate the following iterated integral and sketch the region of integration.

\( \int_0^1 \int_y^1 (x + y) \, dx \, dy \)