Merit Worksheet 2 - Math 242, Fall 2005

1. Calculate the angle \( \theta \) in the following diagram

![Diagram](image)

2. A weight of 125 kilograms is suspended by two cables, as shown in the following diagram.

Find the tension in each cable.

![Diagram](image)

3. Use vectors to show that the line segment joining the midpoint of two sides of a triangle is parallel to the third side and half its length.

4. Prove that the diagonals of a parallelogram \( ABCD \) bisect each other.

5. Find a vector in \( \mathbb{R}^3 \)

   (a) of length 5 in the same direction as \( 12\mathbf{i} + 4\mathbf{j} + 3\mathbf{k} \).
   (b) of length 8 in the opposite direction to \( 2\sqrt{2}\mathbf{i} + 3\mathbf{j} + 2\sqrt{2}\mathbf{k} \).
   (c) of length 1 in the same direction as \( \mathbf{i} + 3\sqrt{7}\mathbf{j} + 15\mathbf{k} \).

6. Do these three points lie on a single straight line?

\[(0, -2, 4), \ (1, -3, 5), \ (4, -6, 8)\]

7. Sketch the graph of \( x^2 + y^2 + z^2 + 8x + 4y - 10z - 4 = 0 \).

8. Let \( \mathbf{a} = 2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}, \ \mathbf{b} = -4\mathbf{i} + 1 + 3\mathbf{k} \) and let \( \mathbf{c} = -4\mathbf{i} + 6\mathbf{j} - 5\mathbf{k} \). Use the Pythagorean Theorem to say whether \( \mathbf{a} \) and \( \mathbf{b} \) are perpendicular. Are \( \mathbf{a} \) and \( \mathbf{c} \) perpendicular?
9. The dot product of two vectors \( \mathbf{a} = a_1 \mathbf{i} + a_2 \mathbf{j} \) and \( \mathbf{b} = b_1 \mathbf{i} + b_2 \mathbf{j} \) in \( \mathbb{R}^2 \) is 
\( a_1 b_1 + a_2 b_2 \). It is written \( \mathbf{a} \cdot \mathbf{b} \).

Let \( \mathbf{a} = -4 \mathbf{i} + 6 \mathbf{j}, \mathbf{b} = 1 \mathbf{i} + 3 \mathbf{j}, \mathbf{c} = 3 \mathbf{i} + 2 \mathbf{j}, \) and \( \mathbf{d} = 6 \mathbf{i} - 2 \mathbf{j} \). Sketch these vectors on a single graph.

(a) Calculate \( \mathbf{a} \cdot \mathbf{b}, \mathbf{a} \cdot \mathbf{c}, \mathbf{b} \cdot \mathbf{c}, \mathbf{b} \cdot \mathbf{d} \).
(b) Do you notice anything?
(c) Find \( \mathbf{b} \cdot \mathbf{b}, \mathbf{c} \cdot \mathbf{c}, \mathbf{a} \cdot \mathbf{a} \).
(d) Find \( ||\mathbf{a}||, ||\mathbf{b}|| \).

10. (Notation you will need for class) Let \( \mathbf{a} = (0, 1) \) and \( \mathbf{b} = (2, -3) \). Sketch the following:

(a) \( S^1(\mathbf{a}, 5) \).
(b) \( B^2(\mathbf{b}, 4) \).
(c) \( B^2(\mathbf{a}, 3) \).
(d) \( D(\mathbf{a}, 3) \).
(e) \( C(\mathbf{b}, 2) \).

**Warm-Up Problems for Thursday**

1. Are \( \mathbf{i} + 2 \mathbf{j} + 3 \mathbf{k} \) and \( -\mathbf{i} + \mathbf{j} - \mathbf{k} \) perpendicular?

2. What are all the vectors parallel to \( \mathbf{i} + 7 \mathbf{j} - 3 \mathbf{k} \)?

3. Let \( \mathbf{a} \) and \( \mathbf{b} \) be as in question 10. What is \( \text{comp}_\mathbf{b} \mathbf{a} \)?