Math 242, Merit Review Questions, Fall 2005

Here are some questions intended as a (partial, incomplete) review of the topics covered this semester. In particular, it only covers Chapters 11 and 12, and NOT chapter 13. Some of these questions are hard. I am also going to give you a photocopy of a list of review questions from a previous year.

1. Write both symmetric and parametric equations for the line that passes through $P_1(1, -1, 2)$ and $P_2(3, 2, -1)$.

2. Given the four points $A(2, 3, 2), B(4, 1, 0), C(-1, 2, 0)$, and $D(5, 4, -2)$, find an equation of the plane that passes through $A$ and $B$ and is parallel to the line through $C$ and $D$.

3. Write an equation for the plane through the point $(1, 1, 1)$ that is normal to the twisted cubic $x = t, y = t^2, z = t^3$ at this point.

4. At time $t = 0$, a ground target is 160 ft from a gun and is moving directly away from it with a constant speed of 80 ft/s. If the muzzle velocity of the gun is 320 ft/s, at what angle of elevation should it be fired in order to strike the moving target?

5. A particle moves in space with parametric equations $x = t, y = t^2, z = \frac{4}{3}t^3$. Find the curvature of its trajectory and the tangential and normal components of its acceleration when $t = 1$.

6. The right branch of the hyperbola $x^2 - y^2 = 1$ may be parametrized by $x(t) = \cosh t, y(t) = \sinh t$. Find the point where its curvature is minimal.

7. Find the vectors (what did he use for notation here?) $\mathbf{N}$ and $\mathbf{T}$ at the point of the curve $x(t) = t \cos t, y(t) = t \sin t$ that corresponds to $t = \pi/2$.

8. Use spherical coordinates to show that

$$\lim_{(x, y, z) \to (0, 0, 0)} \frac{x^3 + y^3 - z^3}{x^2 + y^2 + z^2} = 0$$

9. Prove that there is no function $f$ with continuous second-order partial derivatives such that $f_x(x, y) = 6xy^2$ and $f_y(x, y) = 8x^2y$. 

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10. Write an equation of the plane tangent to the surface

\[ \sin xy + \sin yz + \sin xz = 1 \]

at the point \((1, \pi/2, 0)\).

11. You must build a rectangular shipping crate with volume \(60 \text{ ft}^3\). Its sides cost \$1/\text{ft}^2\), its top costs \$2/\text{ft}^2\), and its bottom costs \$3/\text{ft}^2\). What dimensions would minimize the total cost of the box?

12. Each of the semiaxes \(a, b, c\) of an ellipsoid with volume \(V = \frac{4}{3}\pi abc\) is measured with a maximum percentage error of 1%. Estimate the maximum percentage error in the calculated value of \(V\).

13. Find the point of the surface \(z = xy + 1\) that is closest to the origin.

14. Locate and classify the critical points of the function \(f(x, y) = x^3y^2(1-x-y)\)

15. Find the maximum value of the function \(f(x, y, z) = x + 2y + 3z\) on the curve of intersection of the plane \(x - y + z = 1\) and the cylinder \(x^2 + y^2 = 1\).

16. Find the directional derivative of \(f(x, y) = 2\sqrt{x - y^2}\) at the point \((1, 5)\) in the direction of the point \((4, 1)\). In what direction is the maximum rate of change?

17. If \(z = \cos xy + y \cos x\), where \(x = u^2 + v\) and \(y = u - v^2\), use the chain rule to find \(\frac{\partial z}{\partial u}\) and \(\frac{\partial z}{\partial v}\).