Math 242, Merit Practice Hour Exam 3, Fall 2005

1. Evaluate \( \int_0^8 \int_{x^2/3}^4 x \cos y^4 \, dy \, dx \).

2. Find the volume of the solid bounded by the paraboloids \( z = 2x^2 + 2y^2 \) and \( z = 48 - x^2 - y^2 \).

3. Convert to rectangular coordinates: \( \int_{\pi/2}^{3\pi/2} \int_{1/2}^2 \frac{1}{r} \, dr \, d\theta \).

4. Convert to cylindrical coordinates: \( \int_1^5 \int_{\sqrt{25-x^2}}^{\sqrt{25-x^2}} xy^2 \, dy \, dx \).

5. Apply the first Theorem of Pappus to find the volume of the solid obtained by rotating the region \( L \) around the \( x \)-axis, where \( L \) is the region in the first quadrant bounded by \( x = 1, x = 3 \) and \( y = x^2 \).

6. Apply the second Theorem of Pappus to find the surface area of the surface obtained by rotating the curve \( C \) around the \( y \)-axis, where \( C \) is the semi-circular arc drawn on the board. If \( C \) were more complicated, how would we find its arc-length?

\( (C \) is the right half of the circle of radius 1, centred at (3, 1).\)

7. Set up the integrals but do not evaluate:

Consider a lamina that occupies the region \( D \) bounded by the parabola \( x = 1 - y^2 \) and the coordinate axes in the first quadrant with density function \( \delta(x, y) = y \).

(a) Find the mass of the lamina.

(b) Find the centroid.

(c) Find the moments of inertia and radii of gyration around the \( x \)- and \( y \)-axes.

8. Set up but do not evaluate the triple integrals:

(a) the triple integral for the volume of the solid bounded by the planes \( z = 0, z = 20 - x \) and the cylinder \( x^2 + y^2 = 25 \).

(b) the mass and centroid of a tetrahedron with density \( \delta(x, y, z) = xy + z^2 \), where the tetrahedron lies in the first octant, bounded by the coordinate axes and the plane \( x + y + z = 1 \).

(c) the triple integral for the volume of the solid bounded by \( z = x^2, y + z = 4, y = 0, z = 0 \).