Math 241, Fall 2006, Merit Worksheet 7

1. Consider the curve \( r(t) = \sin t \mathbf{i} + t \mathbf{j} + \cos t \mathbf{k}, 0 \leq t \leq 2\pi. \)
   (a) Find the arc length of \( r(t). \)
   (b) Find the arc length of \( s(t) = \sin t^2 \mathbf{i} + t^2 \mathbf{j} + \cos t^2 \mathbf{k}, 0 \leq t \leq \sqrt{2\pi}. \)
   (c) Find the arc length of \( w(t) = \cos t \mathbf{i} + (t + \frac{\pi}{2}) \mathbf{j} - \sin t \mathbf{k}, -\frac{\pi}{2} \leq t \leq \frac{3\pi}{2}. \)
   (d) Is there a relationship between \( r, s \) and \( w? \)
   (e) What does this tell us about arc length?

2. Find the arc-length parametrization of the helix
   \[ x(t) = 3 \cos t, \quad y(t) = 3 \sin t, \quad z(t) = 4t \]
   in terms of the arc length \( s \) measured from the initial point \((3, 0, 0).\)

3. Sketch or describe the traces of \( x^2 - y^2 + z^2 = 1 \) in both horizontal and vertical planes parallel to the coordinate axes. This is a \textit{hyperboloid of one sheet} opening along the \( y \)-axis.

4. \textbf{True/False}: The equation for the cylinder through the circle \( x^2 + y^2 = 4 \) is \( z = x^2 + y^2 - 4. \)
   What are the horizontal traces and the rulings? What are their equations?

5. Show that the projection into the \( xy \)-plane of the intersection of the plane \( z = y \) and the paraboloid \( z = x^2 + y^2 \) is a circle.

6. Write an equation for the surface generated by rotating the curve \( y^2 + z^2 = 1 \) about the \( z \)-axis. What are its traces parallel to the coordinate planes?

7. Sketch/ describe the traces of \( x^2 - y^2 - 9z^2 = 9. \) This is a \textit{hyperboloid of two sheets} opening along the \( x \)-axis.
Warm-Up Problems for Next Time

1. Find the curvature of

\[ x = 3 \cos t, \quad y = 4 \sin t \]

where \( t = \pi/4 \).

2. Read Section 11.7. Make a list of ALL quadric surfaces. Your list should include the equation, name and a basic sketch of each surface. (This is something you are eventually going to have to memorise).