1. Let \( \mathbf{a}, \mathbf{b}, \mathbf{c} \) be vectors. Then

\[
\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = 
\begin{vmatrix}
  a_1 & a_2 & a_3 \\
  b_1 & b_2 & b_3 \\
  c_1 & c_2 & c_3
\end{vmatrix}
\]

Can you give a reason why \(|\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})| = |(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}|\) holds (without doing any calculations)?

2. (a) Find the parametric and symmetric equations of the line through the points \(P(-1, 0, 2)\) and \(Q(2, 1, 1)\).

(b) Find the midpoint of \(PQ\) and check that it satisfies the equation of the line.

3. Find the equation of the plane through the points \(P(7, 2, 1), Q(6, -1, 3)\) and \(R(9, 3, 2)\).

4. Find the angle \(\theta\) between the planes with equations

\[
3x - 12y + 4z = 12 \quad \text{and} \quad 7x - 4y - z = 11
\]

Write symmetric equations for their line of intersection \(L\).

5. Let the plane \(H\) be given by the equation \(x + y + z = 10\). Write down the equation of a sphere which is tangent to the plane at some point.

6. Show that the points \(A(1, 0, 3), B(3, -2, 9), C(-2, -3, 0), D(4, 1, -1)\) are coplanar.

7. Find the perpendicular distance between the point \((-1, 2, 3)\) and the plane \(3x - 4y + z = 2\).

8. A line \(L\) has symmetric equations

\[
\frac{2x - 5}{1} = \frac{y - 3}{-2} = \frac{z + 4}{3}
\]

(a) By considering just the first inequality what do we obtain?

(b) Similarly find two more. Is there anything special about them?
(c) What is the connection between these and line $L$?

9. What cases can arise when you have two lines $L_1$ and $L_2$ in the $xy$-plane? What does this tell us about two linear equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ in two unknowns $x$ and $y$?

10. What are the possible configurations for

(a) Three lines in $\mathbb{R}^3$?

(b) Three planes in $\mathbb{R}^3$?

What does this tell you about solutions of suitable systems of linear equations?

Warm-Up Problems for Next Time

1. Show that the graph of the curve with parametric equations $x = t \sin 6t, y = t \cos 6t, z = t$ lies on the cone $z = \sqrt{x^2 + y^2}$ with its vertex at the origin and opening upwards.

2. Suppose $\mathbf{r}(t) = 3\mathbf{i} \cos 2\pi t + 3\mathbf{j} \sin 2\pi t$. Find $\mathbf{r}'(7/4), \mathbf{r}''(7/4)$. 