Math 241, Fall 2006, Merit Worksheet 3

1. You are given a line \( ax + by + c = 0 \). Find a normal vector \( \vec{n} \) to this line.

   Hint: If \( P_0 = (x_0, y_0) \) and \( P_1 = (x_1, y_1) \) are points on the line, then \( \overrightarrow{P_0P_1} \) is . . .

   This will be useful in one of your homework problems.

2. Calculate \((-i + 4j + k) \times (3i + 2j - k)\).

3. Find the area of the triangle in \( \mathbb{R}^3 \) between the points \( A = (1, 1, 1) \), \( B = (4, 5, 2) \) and \( C = (6, 3, 8) \).

4. The cross product:

   (a) Consider the vector \( \vec{c} = \vec{a} \times (\vec{a} \times \vec{b}) \). Is \( \vec{c} \perp \vec{a} \)? Is \( \vec{c} \perp \vec{b} \)? Prove or give a counterexample.

   (b) Suppose that \( j \times \vec{a} = i \). Give two possible solutions for \( \vec{a} \) and discuss the other possible solutions.

5. When are two vectors parallel? Draw pictures that explain and also make up a (nontrivial) example of two vectors that are parallel and two vectors that are not parallel.

6. When are two vectors perpendicular? Draw pictures and work out examples.

7. Given the points \( A = (1, -2, 7) \) and \( P = (x, y, z) \) and the vector \( \vec{n} = <1, -5, 3> \):

   (a) Write an equation that says the following: \( \vec{n} \) and \( \overrightarrow{AP} \) are perpendicular.

   (b) Describe the set of points \( P(x, y, z) \) that satisfy the equation from (a).

8. A parallelepiped is a 3-d object with parallelograms for side, like a tilted box. Its volume is the area of its base times its height (in the direction perpendicular to that base). The three vectors \( \vec{a} = 6i + 2j + k \), \( \vec{b} = i + 5j \), \( \vec{c} = 2i + 2j + 7i \) determine a parallelepiped. Find its volume.

   Find a formula for the volume of a parallelepiped with sides \( \vec{x}, \vec{y}, \vec{z} \).
9. (a) Find the parametric and symmetric equations of the line through the points \(P(-1, 0, 2)\) and \(Q(2, 1, 1)\).

(b) Find the midpoint of \(PQ\) and check that it satisfies the equation of the line.

10. Find the equation of the plane through the points \(P(7, 2, 1)\), \(Q(6, -1, 3)\) and \(R(9, 3, 2)\).

**Warm-Up Problems for Next Time**

1. Find the equation of the plane through \((1, -3, 2)\) with normal vector \(2i + 3j - 4k\).

2. Find \(r'(0)\) for \(r(t) = e^{2t}i + e^{-t}j\).