Math 241 Fall 2006, Merit Worksheet 18

1. Suppose that you are standing at the point with coordinates \((-100, -100, 430)\) on a hill that has the shape of the graph of \(z = 500 - 0.003x^2 - 0.004y^2\) (in units of metres). In what (horizontal) direction should you move in order to maintain a constant altitude?

2. Use the contour plot on the next page (it shows the level curves of a surface) to answer the following questions: At which point will the gradient vector have the largest magnitude? At which of these points will the gradient vector be most parallel to \(\vec{j}\)?

   (a) \((0, 4)\)
   (b) \((-4, -4)\)
   (c) \((0, 0)\)
   (d) \((6, -2)\)

3. Use the gradient vector to write an equation for the line tangent to the curve \(x^4 + xy + y^2 - 19 = 0\) at the point \((2, -3)\).

4. The surfaces \(x^2y^2 + ax + z^3 = 16\) and \(3x^2 + y^2 - 2z = 9\) intersect in a curve that passes through the point \(P(2, 1, 2)\). Find a tangent vector to the curve of intersection at \(P\).

5. Find an equation for the plane tangent to the paraboloid \(z = 2x^2 + 3y^2\) and, simultaneously, parallel to the plane \(4x - 3y - z = 10\).

6. The curve \(\mathbf{r}(t) = <t^2/2, 4/t, t/2 - t^2>\) intersects the surface \(x^2 - 4y^2 - 4z = 0\) at the point \((2, -2, 3)\). What is the angle of intersection?

7. Consider the function \(f(x, y) = \frac{1}{2}(ax^2 + by^2)\).

   (a) Show that \((0, 0)\) is a critical point.
   (b) For what values of \(a\) and \(b\) does \(f\) have a maximum at \((0, 0)\)? What does the surface look like?
   (c) For what values of \(a\) and \(b\) does \(f\) have a minimum at \((0, 0)\)? What does the surface look like?
   (d) For what values of \(a\) and \(b\) does \(f\) have a saddle point at \((0, 0)\)? What does the surface look like?
8. Which of the following guarantees a saddle point of the function $f(x, y)$ at the point $(a, b)$?

(a) $f_{xx}$ and $f_{yy}$ have the same sign at $(a, b)$
(b) $f_{xx}$ and $f_{yy}$ have different signs at $(a, b)$
(c) $f_{xy}$ is negative at $(a, b)$
(d) None of the above.

9. Which of the following would be enough evidence to conclude that $f(x, y)$ has a global minimum?

(a) $D$ is always positive
(b) $f_{xx} > 0$ and $f_{yy} > 0$
(c) $f(x, y)$ has no saddle point and no local maxima
(d) None of the above.

**Warm-Up for Next Time**

1. Find and classify the critical points of the function

$$f(x, y) = x^3 + y^3 + 3xy + 3$$