Math 241 Fall 2006, Merit Worksheet 16

1. Find the first octant point \( P(x, y, z) \) on the plane \( 2x + 3y + z = 49 \) which is closest to the point \( Q(7, -7, 0) \).

2. Find the maximum possible product of three positive numbers whose sum is 120.

3. Use the example \( x^2 + y^2 = 1 \) to explain (in detail) the implicit function theorem.

4. Use implicit differentiation to find \( z_x \) and \( z_y \) where \( z \) is the function implicitly defined by \( x - yz + xy^2z^3 = 1 \) at the point \( (1, 1, 1) \).

5. A particle \( Q \) moving through space is being studied. Let \( s \) denote the distance that \( Q \) has travelled with respect to some starting point. (We can think of distance as arc length on the curve determined by \( Q \).)

   (a) We know that \( S \) depends on two factors \( X \) and \( Y \).

   (b) We know that \( X \) and \( Y \) vary over time \( t \) in years according to the formulae \( X = t^2 - 1 \) and \( Y = \ln t \).

   (c) The changes in \( s \) with respect to \( X \) and \( Y \) are both constants, \( a \) and \( b \) respectively.

   What is the speed of the particle \( Q \), in terms of \( a \) and \( b \), after 20 years?

6. Suppose \( R = f(u, v, w), \) \( u = g(x, y, z), \) \( v = h(x, y, z) \) and \( w = j(x, y, z) \). In the chain rule, how many terms do you have to add up to find the partial derivative with respect to \( x \)?

7. Suppose \( w = \ln(x^2 + y^2 + z^2), \) where \( x = s - t, \) \( y = s + t \) and \( z = 2\sqrt{st}. \) Find \( \frac{\partial w}{\partial s} \) and \( \frac{\partial w}{\partial t}. \)

8. If \( g(s, t) = f(s^2 - t^2, t^2 - s^2) \) and \( f \) is differentiable, show that \( g \) satisfies the partial differential equation

\[
\frac{t}{\partial s} + s \frac{\partial g}{\partial t} = 0
\]
9. The radius of a right circular cylinder is decreasing at a rate of 1.5 cm/s while its height is increasing at a rate of 4 cm/s. At what rate is the volume of the cylinder changing when the radius is 50 cm and the height is 100 cm? The surface area?

10. If $f(u, v) = 2u^2v$ and $u(x, y) = x + 2y$ and $v = x^2 - y$, calculate $f_{xx}$.

11. Change variables in the partial differential equation $z_{xx} - z_{yy} - z_{xy} = 0$ if $u = x^2 + y^2$, $v = 2xy$.

**Warm-Up for Next Time**

1. Find the gradient vector to the function $f(x, y, z) = y^2 - z^2$ at the point $P(17, 3, 2)$. 