Math 242, Merit Review Questions for Hour Exam 3, Fall 2006

Usual warnings apply: I am not writing the exam and I have not seen the exam. I would also suggest looking at the homework problems, especially the extra problems posted online. You should also look at the list of topics for exam 3 that Professor Nikolaev has posted online. Tuesday morning will be a review session for the exam.

1. Find the absolute maximum and minimum values that the function \( f(x, y, z) = x^2 - yz \) takes on the region \( x^2 + y^2 + z^2 \leq 1 \).

2. Evaluate the integral
   \[
   \int_{0}^{4} \int_{\sqrt{y}}^{2} \frac{ye^{x^2}}{x^3} \, dx \, dy.
   \]

3. Find the area of that section of the saddle-shaped surface \( z = xy \) inside the cylinder \( x^2 + y^2 = 1 \).

4. Substitute \( u = xy \) and \( v = xy^3 \) to find the area of the first quadrant region bounded by the curves \( xy = 2 \), \( xy = 4 \), \( xy^3 = 3 \), \( xy^3 = 6 \). (Fig. 13.9.8, p.1010)

5. Convert to rectangular coordinates:
   \[
   \int_{\pi/2}^{3\pi/2} \int_{0}^{1} \frac{1}{r} \, dr \, d\theta.
   \]

6. Convert to cylindrical coordinates:
   \[
   \int_{1}^{5} \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} xy^2 \, dy \, dx.
   \]

7. Set up the integrals but do not evaluate:
   Consider a lamina that occupies the region \( D \) bounded by the parabola \( x = 1 - y^2 \) and the coordinate axes in the first quadrant with density function \( \delta(x, y) = y \).
   
   (a) Find the mass of the lamina.
   (b) Find the centroid.
   (c) Find the moments of inertia around the \( x \)- and \( y \)-axes.

8. Set up but do not evaluate the triple integrals:
(a) the triple integral for the volume of the solid bounded by the planes \( z = 0, z = 20 - x \) and the cylinder \( x^2 + y^2 = 25 \).

(b) the mass and centroid of a tetrahedron with density \( \delta(x, y, z) = xy + z^2 \), where the tetrahedron lies in the first octant, bounded by the coordinate axes and the plane \( x + y + z = 1 \).

(c) the triple integral for the volume of the solid bounded by \( z = x^2 \), \( y + z = 4 \), \( y = 0 \), \( z = 0 \).

9. Calculate the divergence and curl of the vector field:

\[
\mathbf{F}(x, y, z) = xy^2 \mathbf{i} + yz^2 \mathbf{j} + zx^2 \mathbf{k}.
\]

10. Evaluate the line integral along the curve \( y = x^3 \) as \( x \) goes from \( x = 3 \) to \( x = 0 \).

\[
\int_C xy^2 \, dx + xy \, dy.
\]

11. Evaluate the arclength integral along the curve \( y = x^3 \) as \( x \) goes from \( x = 3 \) to \( x = 0 \).

\[
\int_C y \, ds.
\]

12. Evaluate

\[
\int_C F_t \, ds
\]

where \( \mathbf{F}(x, y, z) = y\mathbf{i} - x\mathbf{j} + z\mathbf{k} \) and \( C \) is the curve parametrized by \( x = \sin t, y = \cos t, z = 2t \), for \( 0 \leq t \leq \pi \).