Prepare tabs:
   (1) Lecture notes.
   (2) plot3d $y^2$.
   (3) Announcements.
   (4) Graph $x^2 + y^2 - z^2 = 0$.
   (5) Graph $x^2 + y^2 - z^2 = 1$.
   (6) Graph $x^2 + y^2 - z^2 = -1$.
   (7) $x^2 + 4y^2 + 9z^2 = 1$.

Begin on tab (1).

Previously

Question 1.
Draw or construct the graph $z = y^2$
Are you done?
(A) Yes
(B) Not yet

Click B. Switch to A when you finish.

If you are done, sketch the contour plot & check with your neighbor.
Show tab (2).

$z = y^2$ is a **parabolic cylinder**.

A **cylinder** is a surface consisting of all lines parallel to a given line and passing through a given plane curve.

Tip: One variable missing $\implies$ the surface is a cylinder.

Show tab (3). Read announcements.

**Example.**

*Draw the level sets of* $f(x, y, z) = x^2 + y^2 - z^2$.

$x = 0 \& x^2 + y^2 - z^2 = k \implies y^2 - z^2 = k$.

We drew these level sets last time.

Make & label contour map of $y^2 - z^2$.

Let $r = \sqrt{x^2 + y^2}$ be the distance from the $z$ axis.

Then $f(x, y, z) = r^2 - z^2$.

$\implies$ each level set is symmetric about the $z$-axis.
\[ x^2 + y^2 - z^2 = k \]
is:

- a **cone** if \( k = 0 \),

Draw a cone.

- a **hyperboloid of one sheet** if \( k > 0 \), and

Draw a hyperboloid of one sheet.

- a **hyperboloid of two sheets** if \( k < 0 \).

Draw a hyperboloid of two sheets.

Show tabs (4), (5) & (6).
Example. Draw the level sets of

\[ f(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}. \]

\[ z = 0 \& \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = k: \]
\[ \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = k. \]
\[ \Rightarrow \text{ellipse w/ intercepts } (\pm a\sqrt{k}, 0) \& (0, \pm b\sqrt{k}) \text{ if } k > 0. \]

Sketch ellipse

Similar claims hold for \( x = 0 \& y = 0 \)
\[ \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = k \text{ is} \]
• empty if \( k < 0 \),
• \((0, 0, 0)\) if \( k = 0 \& \)
• an ellipsoid if \( k > 0 \),
 w/ intercepts \((\pm a\sqrt{k}, 0, 0), (0, \pm b\sqrt{k}, 0) \& (0, 0, \pm c\sqrt{k})\).

Sketch ellipsoid

Show tab (7).
A **quadric surface** is the zero set of a polynomial of degree two in three variables, e.g.

\[ Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0. \]

Skip fact if less than 20 minutes remaining.

**Fact:** every quadric surface is either

- an ellipsoid,
- an elliptic paraboloid,
- a hyperbolic paraboloid,
- a cone,
- a hyperboloid of one or two sheet, or
- a cylinder (of some type).
LIMITS IN ONE VARIABLE

Suppose we:

- **Want** a square w/ area 1.
- **Get** a square w/ side length $1 + h$.

**Draw square of side length $1 + h$ and shade error.**

\[
\text{Error} = \text{area} - 1
\]
\[
\Rightarrow E(h) = (1 + h)^2 - 1 = h^2 + 2h.
\]

Sketch $E(h)$ over $(-1, \infty)$.

We say $\lim_{h \to 0} E(h) = 0 \iff$

Given any $\epsilon > 0$, we can find $\delta > 0$ so that

\[
0 < |h| < \delta \Rightarrow |E(h)| < \epsilon.
\]

$\iff$ You can satisfy every customer, no matter how picky.
View as challenge-response.

Given $\epsilon = \frac{1}{10}$?

Let $\delta = \frac{1}{30}$.

$$0 < |h| < \delta = \frac{1}{30}$$

$$\Rightarrow |E(h)| = |h^2 + 2h| \leq |h|^2 + 2|h| \leq \frac{1}{30} \cdot 30 + \frac{2}{30} < \frac{1}{10} = \epsilon.$$ 

What about $\delta = \frac{1}{100}$?

Also OK, because

$$\frac{1}{100} \cdot 100 + \frac{2}{100} < \frac{1}{10}.$$ 

What about $\delta = \frac{1}{10}$?

Not OK, because $0 < |\frac{1}{20}| < \frac{1}{10}$ but

$$\left| E \left( \frac{1}{20} \right) \right| = \frac{1}{20} \cdot 20 + \frac{2}{20} > \frac{1}{10}.$$ 

Tip: Use any $\delta$ that works.

Use these choices for all remaining questions.

**Question 2.** What $\delta$ works for $\epsilon = \frac{1}{100}$?

(1) $\frac{1}{100}$
(2) $\frac{1}{300}$
(3) $\frac{1}{1000}$
(4) Some other $\delta$.
(5) No value will work.

**Question 3.** What $\delta$ works for $\epsilon = \frac{1}{1000}$?
We can always find some $\delta$ to satisfy every customer!

If less than 5 minutes remain, skip to next example

For small $\epsilon > 0$, let $\delta = \frac{\epsilon}{3}$.

$$0 < |h| < \delta = \frac{\epsilon}{3}$$

$$\Rightarrow |E(h)| = \leq |h^2| + 2|h| \leq \frac{\epsilon \cdot \epsilon}{3 \cdot 3} + \frac{2\epsilon}{3} < \epsilon.$$

**Example.** Now suppose $E(h) = \frac{1}{20} \sin \frac{1}{h}$.

Is $\lim_{h \to 0} E(h) = 0$?

**Graph $E(h)$ over $(-1, \infty)$.**

**Question 4.** What $\delta$ works for $\epsilon = \frac{1}{10}$?

**Question 5.** What $\delta$ works for $\epsilon = \frac{1}{30}$?

We say that $\lim_{x \to a} f(x) = c \iff f(a + h) = c + E(h)$, where $\lim_{h \to 0} E(h) = 0$.  