Question 1.

Let \( C \) be parameterized by \( \mathbf{r}(t) = \langle t, 2t \rangle, \) \( 0 \leq t \leq 1. \)

If \( \mathbf{F}(x, y) = \langle 1, 2y \rangle, \) \( \int_C \mathbf{F} \cdot d\mathbf{r} \) is

(A) \(-10\)
(B) \(-5\)
(C) \(0\)
(D) \(5\)
(E) \(10\)

If you’re done: Sketch \( C \) & \( \mathbf{F} \). Is your answer reasonable?

Show tab 2 & read announcements.
Oriented curves 16.2

For today:
- Curves are smooth,
- vector fields continuous &
- functions differentiable.

Let $C$ be parameterized by $\mathbf{r}(t), a \leq t \leq b$. Let $\mathbf{T} = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$ be the unit tangent vector.

**Theorem.**
$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \mathbf{F} \cdot \mathbf{T} ds$$

**Proof.**
$$\int_C \mathbf{F} \cdot \mathbf{T} ds = \int_a^b \mathbf{F} \cdot \mathbf{T} |\mathbf{r}'| dt$$
$$= \int_a^b \mathbf{F} \cdot \frac{\mathbf{r}'}{|\mathbf{r}'|} |\mathbf{r}'| dt$$
$$= \int_a^b \mathbf{F} \cdot \mathbf{r}' dt = \int_C \mathbf{F} \cdot d\mathbf{r}.$$
You can travel two possible directions on any curve; these are called **orientations**

On left, draw two copies of a curve w/ endpoints $P \neq Q$. Show both orientations & write:

From $P$ to $Q$ **or** from $Q$ to $P$.

On right, repeat w/ two copies of a circle &:

Counterclockwise **or** clockwise.

Let $C$ be an oriented curve parameterized by $\mathbf{r}(t)$ & $-C$ be the same curve with the opposite orientation.

- For a different parameterization of $C$ (w/ the **same** orientation), the unit tangent vector is still $\mathbf{T} \Rightarrow \int_C \mathbf{F} \cdot d\mathbf{r}$ doesn’t depend on the parameterization.
- For a parameterization of $-C$ (w/ the **opposite** orientation), the unit tangent vector is $-\mathbf{T} \Rightarrow$

\[
\int_{-C} \mathbf{F} \cdot d\mathbf{r} = \int_{-C} \mathbf{F} \cdot \mathbf{T} ds = \int_C \mathbf{F} \cdot (-\mathbf{T}) ds =
\]

\[
- \int_C \mathbf{F} \cdot \mathbf{T} ds = - \int_C \mathbf{F} \cdot d\mathbf{r}.
\]
Question 2. Let $C$ be $x^2 + y^2 = 1$, oriented clockwise.

Sketch $C$ & $F$.

Use $\int_C F \cdot dr = \int_C F \cdot T \, ds$ to find $\int_C \langle -y, x \rangle \cdot dr$.

(A) $-2\pi$
(B) $-\pi$
(C) 0
(D) $\pi$
(E) $2\pi$
**Fundamental Theorem of Line integrals [16.3].**

Fundamental Theorem of Calculus: 
Given \( f : [a, b] \to \mathbb{R} \), 
\[
\int_a^b f'(t)dt = f(b) - f(a).
\]

**Theorem** (Fundamental Theorem of Line Integrals). 
Let \( C \) be a curve from \( P \) to \( Q \). Given \( g : C \to \mathbb{R} \), 
\[
\int_C \nabla g \cdot d\mathbf{r} = g(Q) - g(P)
\]

**Proof.** Let \( \mathbf{r}(t) = \langle x(t), y(t) \rangle \), \( a \leq t \leq b \), parameterize \( C \). 
\[
\int_C \nabla g \cdot d\mathbf{r} = \int_a^b \nabla g(\mathbf{r}(t)) \cdot \mathbf{r}'(t)dt
\]
\[
= \int_a^b \langle g_x, g_y \rangle \cdot \langle x', y' \rangle dt
\]
\[
= \int_a^b (g_x x' + g_y y') dt
\]
\[
= \int_a^b \frac{d}{dt} (g(x(t), y(t))) dt
\]
\[
= g(x(t), y(t)) \bigg|_a^b
\]
\[
= g(Q) - g(P)
\]
Example.
Let $C$ be the line segment from $(0, 0)$ to $(1, 2)$.
Let $g = x + y^2$ & find $\int_C \nabla g \cdot d\mathbf{r}$.

$$\int_C \nabla g \cdot d\mathbf{r} = g(1, 2) - g(0, 0) = (1 + 2^2) - (0 + 0^2) = 5.$$ 
Note: Since $\nabla g = \langle 1, 2y \rangle$, this is Question 1.

Ask one or both of these questions if time permits.

Recall that $\mathbf{F}$ is conservative if $\mathbf{F} = \nabla g$ for some $g$.

Show tab 3.

Question 3. Is $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for every circle $C$?
(A) My partner and I agree that it is.
(B) We agree that it isn’t.
(C) We don’t agree.

Question 4. Is $\langle -y, x \rangle$ conservative?
(A) My partner and I agree that it is.
(B) We agree that it isn’t.
(C) We don’t agree.

Hint: Use the Fundamental Theorem of Line Integrals.