1. $\kappa'(G) = \delta(G)$ for diameter 2. Let $G$ be a simple graph with diameter 2, and let $[S, \overline{S}]$ be a minimum edge cut with $|S| \leq |\overline{S}|$.
   a) Prove that every vertex of $S$ has a neighbor in $\overline{S}$.
   b) Use part (a) and Corollary 4.1.13 to prove that $\kappa'(G) = \delta(G)$.

2. (The block-cutpoint graph). Let $H$ be the block-cutpoint graph of a graph $G$ that has a cut-vertex.
   a) Prove that $H$ is a forest.
   b) Prove that $G$ has at least two blocks each of which contains exactly one cut-vertex of $G$.
   c) Prove that a graph $G$ with $k$ components has exactly $k + \sum_{v \in V(G)} (b(v) - 1)$ blocks, where $b(v)$ is the number of blocks containing $v$.
   d) Prove that every graph has fewer cut-vertices than blocks.

3. Let $G$ be a 2-connected graph. Prove that if $T_1$ and $T_2$ are two spanning trees of $G$, then $T_1$ can be transformed into $T_2$ by a sequence of operations in which a leaf is removed and reattached using another edge of $G$.

4. Suppose that $\kappa(G) = k$ and $\text{diam } G = d$. Prove that $\eta(G) \geq k(d - 1) + 2$ and $\alpha(G) \geq \lceil (1 + d)/2 \rceil$. For each $k \geq 1$ and $d \geq 2$, construct a graph for which equality holds in both bounds (simultaneously).

5. Let $X$ and $Y$ be disjoint sets of vertices in a $k$-connected graph $G$. Let $u(x)$ for $x \in X$ and $w(y)$ for $y \in Y$ be nonnegative integers such that $\sum_{x \in X} u(x) = \sum_{y \in Y} w(y) = k$. Prove that $G$ has $k$ pairwise internally disjoint $X,Y$-paths so that $u(x)$ of them start at $x \in X$ and $w(y)$ of them end at $y \in Y$.

6. Prove that applying the expansion operation of Example 1.3.26 to a 3-connected graph yields a 3-connected graph. Obtain the Petersen graph from $K_4$ by expansions. (Comment: Tutte prove that a 3-regular graph is 3-connected if and only if it arises from $K_4$ by a sequence of these operations.)