HOMEWORK I

1. Consider the group $\mathbb{Q}/\mathbb{Z}$. Let $c_m = \text{image of } \frac{1}{m} \text{ in } \mathbb{Q}/\mathbb{Z}$ ($m > 0$) and $\langle c_m \rangle$ denote the cyclic subgroup generated by $c_m$.
   a) Show $\ldots \langle c_m \rangle \subsetneq \langle c_{m+1} \rangle \subsetneq \ldots$.
   b) Let $\mathbb{Z}(p^\infty) = \bigcup_{m=1}^{\infty} \langle c_m \rangle$. Show that every proper subgroup of $\mathbb{Z}(p^\infty)$ is of the form $\langle c_m \rangle$ for some $m > 0$.
   c) Show that $\mathbb{Z}(p^\infty)/(c_m) \cong \mathbb{Z}(p^\infty)$.

2. Let $G$ be a group and let $Z(G)$ denote its center. Prove that if $G/Z(G)$ is cyclic then $G$ is abelian.

3. Let $G = A_4$ - the normal subgroup consisting of even permutations of $S_4$. Show that $G$ cannot have a subgroup of order 6.
   **Hint:** If such a subgroup $H$ exists, then show that $H$ is normal and for every $\sigma \in G$, $\sigma^2 \in H$. 