1. Let \( f(x, y) = xy + x - 2y - 2 \), where \( x(t) = 1 + t, y(t) = 2 - t \). Compute \( \frac{df}{dt}(1) \).

2. Let \( f(x, y) = xy \), where \( x(s, t) = s + t, y(s, t) = s - t \). Compute \( \frac{\partial f}{\partial t}(1, 1) \).

3. Suppose that \( z = x^2 + y^3 \), where \( x = st \) and \( y \) is a function of \( s \) and \( t \). Suppose further that when \( (s, t) = (2, 1) \), \( \frac{\partial y}{\partial t} = 0 \). Determine \( \frac{\partial z}{\partial t}(2, 1) \).

4. Suppose that \( z = f(x, y) \) has continuous partial derivatives. Let \( x = e^r \cos \theta, y = e^r \sin \theta \). Show that then

\[
\left( \frac{\partial z}{\partial x} \right)^2 + \left( \frac{\partial z}{\partial y} \right)^2 = e^{-2r} \left[ \left( \frac{\partial z}{\partial r} \right)^2 + \left( \frac{\partial z}{\partial \theta} \right)^2 \right]
\]

5. If \( \omega = g(u^2 - v^2, v^2 - u^2) \) has continuous partial derivatives with respect to \( x = u^2 - v^2 \) and \( y = v^2 - u^2 \), show that

\[
v \frac{\partial \omega}{\partial u} + u \frac{\partial \omega}{\partial v} = 0
\]

6. Suppose that \( y \) is defined implicitly as a function \( y(x) \) by an equation of the form

\[ F(x, y) = 0 \]

(For example, the equation \( x^3 - y^2 = 0 \) defines as two functions of \( x \), namely \( y = x^{3/2} \) and \( y = -x^{3/2} \). The equation \( \sin(xy) - x^2y^7 + e^y = 0 \), on the other hand, cannot readily be solved for \( y \) in terms of \( x \), and hence only defined implicitly.)

(a) Show that if \( F \) and \( y(x) \) are both assumed to be differentiable, then

\[
\frac{dy}{dx} = \frac{F_x(x, y)}{F_y(x, y)} = -\frac{\partial F}{\partial x} \left/ \frac{\partial F}{\partial y} \right.
\]

provided \( F_y(x, y) \neq 0 \). (Hint: use the chain rule to differentiate \( F(x, y(x)) \) with respect to \( x \).)

(b) Use the result of part (a) to find \( \frac{dy}{dx} \) when \( y \) is defined implicitly in terms of \( x \) by the equation \( x^3 - y^2 = 0 \). Check your result by explicitly solving for \( y \) and differentiating.