1. Given a curve \( \mathbf{r}(t) = (e^t \cos t, 0, e^t \sin t) \), \( t \in (-\infty, 2\pi] \).
   
   (1) Briefly sketch the curve.
   
   (2) Compute \( \mathbf{r}'(t) \) and \( \|\mathbf{r}'(t)\| \).
   
   (3) Compute the arc-length for \( t \in (-\infty, 2\pi] \).

2. In this exercise, we will prove the theorem: If \( \|\mathbf{r}(t)\| \) is constant for all \( t \), then \( \mathbf{r}(t) \perp \mathbf{r}'(t) \). To do that, follow the following instructions:
   
   (1) Square the equation \( \|\mathbf{r}(t)\| = c \) and write it as a dot product.
   
   (2) Differentiate both side with respect to \( t \). You need to use the product rule.
   
   (3) Simplify the equation. What conclusion can you make?

Now consider the circular motion \( \mathbf{r}(t) = (\cos t, \sin t) \). We have seen that \( \mathbf{r}'(t) \) is the velocity. Analogously, we can realize \( \mathbf{r}''(t) \) as the acceleration. Show that \( \mathbf{r}'(t) \perp \mathbf{r}''(t) \).

3. In this problem, we will define re-parametrization.
   
   (1) Notice that parametrization to a curve is never unique. Compare the curve \( \mathbf{r}_1(t) = (\cos t, \sin t) \) and \( \mathbf{r}_2(t) = (\cos 2t, \sin 2t) \), where \( t \in [0, 2\pi] \) for both curves. Do they have the same image set? How many time does each of them wrap around the origin? Compute the speed of each curve.

Given an increasing function or a decreasing function \( t = f(\theta) \) (we use smooth monotone function because we want time to go only in one direction), we define the parametrization of a curve \( \mathbf{r}(t) \) to be \( \mathbf{r}(f(\theta)) \).

   (2) Given a curve \( \mathbf{r}(t), t \in [a, b] \) and an increasing function \( f(\theta), \theta \in [c, d] \) such that \( f(c) = a, f(d) = b \). Does \( \mathbf{r}(f(\theta)) \) have the same image as the one of \( \mathbf{r}(t) \)? Do they have the same direction?

   (3) Given a curve \( \mathbf{r}(t), t \in [a, b] \) and a decreasing function \( f(\theta), \theta \in [c, d] \) such that \( f(c) = b, f(d) = a \). Does \( \mathbf{r}(f(\theta)) \) have the same image as the one of \( \mathbf{r}(t) \)? Do they have the same direction?

   (4) Compute the speed of the curve \( \mathbf{r}(f(\theta)) \).