1. Let \( C \) be the parameterized curve \( \mathbf{r}(t) = \langle \cos t, \sin 2t, \sin t \rangle, \ 0 \leq t \leq 2\pi \).

(a) Find an oriented surface in \( \mathbb{R}^3 \) which has the curve \( C \) as its boundary. Hint: find a relationship between \( x, y, z \).

(b) Use Stoke's theorem to compute \( \int_C \mathbf{F} \cdot d\mathbf{r} \), where \( \mathbf{F} = \langle xy + x^3, 2z, 3y \rangle \).

2. Consider the torus \( T \) parameterized by
\[
(x, y, z) = \left( (2 + \cos \theta) \cos \psi, (2 + \cos \theta) \sin \psi, \sin \theta \right), \quad 0 \leq \theta, \psi \leq 2\pi.
\]
This problem will take you step by step through one approach to problem 6 on Tuesday's worksheet (which I didn't expect you to have enough time to get to last time).

(a) Draw a picture of \( T \). [Hint: \( \theta \) is the same \( \theta \) from cylindrical coordinates, and \( \psi \) is the angle "on the inside" of \( T \).]

(b) In the next few steps, we construct a vector field on \( \mathbb{R}^3 \) which gives normal vectors when restricted to \( T \). Draw the cross section of \( T \) in the \( xz \)-plane. Your drawing should include a circle of radius 1 centered at \( (x, z) = (2, 0) \).

(c) Which other cross sections of \( T \) will look the same? Label the \( x \)-axis "\( r \)".

(d) Write down a translation \( \mathbf{F}_0 \) of the vector field \( \langle r, z \rangle \) on the \( rz \)-plane which gives unit normals to the circle mentioned in part (b).

(e) Rotate the vector field \( \mathbf{F}_0 \) about the \( z \)-axis to get a vector field \( \mathbf{F} \) on \( \mathbb{R}^3 \). The vector field \( \mathbf{F} \) will not be defined on the \( z \)-axis, but that is okay. It may help to think about this part in cylindrical coordinates, and then change the result back to cartesian coordinates.

(f) Compute \( \text{div} \, \mathbf{F} \). To do this, you should either make sure that \( \mathbf{F} \) is written all in terms of \( x, y, z \), or use the chain rule carefully.

(g) Parameterize the solid region \( E \) inside of \( T \). One way to do this is by modifying the parameterization of \( T \) above to include another parameter measuring the distance from the circle that runs right down the middle of the torus. Another is to use cylindrical coordinates.

(h) Find the Jacobian of the transformation constructed in part (g).

(i) Compute
\[
\text{Area}(T) = \iint_T \mathbf{n} \cdot d\mathbf{S} = \iiint_E \text{div} \, \mathbf{F} \, dV.
\]