1. Let $S$ denote the unit sphere. The surface area of $S$ may be computed as the flux of the normal vector field $\mathbf{n}$ field through $S$ as follows:

$$\text{Area}(S) = \iint_S 1 \, dS = \iint_S \mathbf{n} \cdot \mathbf{n} \, dS.$$ 

Use the Divergence Theorem to compute $\text{Area}(S)$ as a triple integral.

2. Let $S$ be the portion of the cylinder of radius 2 about the $x$-axis where $-1 \leq x \leq 1$.

(a) Draw a picture of $S$ and compute its area without doing any integrals.

(b) Find a parameterization $r(u, v)$ of $S$.

(c) Does the normal vector field associated to your parameterization point into or out of $S$? First try to determine this without doing any calculations, and then check your answer by evaluating $r_u \times r_v$.

(d) If necessary, change your parameterization so that the normal vector field points inwards.

(e) Now consider the vector field $\mathbf{F} = \langle -z, xz, -xy \rangle$. Compute $\text{curl} \mathbf{F}$.

(f) Check that $\text{curl} \mathbf{F}$ is the sum of $G = \langle -2x, -1, 0 \rangle$ and $H = \langle 0, y, z \rangle$.

(g) Use geometric arguments to determine whether the flux of $G$ is positive, zero, or negative. Remember that we have oriented $S$ so that the normals point inwards. Do the same for $\text{curl} \mathbf{F}$.

(h) Using your parameterization, directly compute the flux of $\text{curl} \mathbf{F}$.

(i) Check your answer in (h) using Stokes’ Theorem. Note here that $\partial S$ has two boundary components.

(j) Check your answer in (h) a second time by using what you learned in (g) to compute the flux of $G$ and $H$.

3. Let $\mathbf{F} = \langle y^2, x^2, z^2 \rangle$. Show that

$$\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \int_{C_2} \mathbf{F} \cdot d\mathbf{r}$$

for any two closed curves as shown lying on a cylinder about the $z$-axis.
4. Consider the surface $S$ shown below, which is oriented using the outward pointing normal.

(a) Suppose $F$ is a vector field on $\mathbb{R}^3$ which is equal to curl $G$ for some unknown vector field $G$. Suppose the line integral of $G$ around the unit circle (oriented counter-clockwise) in the $xy$-plane is 25. Determine the flux of $F$ through $S$.

(b) Suppose $H$ is a vector field on $\mathbb{R}^3$ which is equal to curl $B$ for some unknown vector field $B$. If $H(x, y, 0) = \mathbf{k}$, find the flux of $H$ through the surface $S$.

5. Consider the surface $T$ which is the intersection of the plane $x + 2y + 3z = 1$ with the first octant.

(a) Draw a picture of $T$.

(b) Use Stokes’ Theorem to evaluate $\int_{\partial T} F \cdot dr$ for $F = \langle y, -2z, 4x \rangle$. Here, you should orient $\partial T$ counterclockwise when viewed from $(2, 2, 2)$.

6. If time remains, repeat problem 1 for the torus parameterized by

$$(x, y, z) = \left((2 + \cos \theta) \cos \phi, (2 + \cos \theta) \sin \phi, \sin \theta \right), \quad 0 \leq \theta, \phi \leq 2\pi.$$