1. Consider the ellipsoid with implicit equation
\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1. \]
(a) Parameterize this ellipsoid.
(b) Set up, but do not evaluate, a double integral that computes its surface area.

2. Let \( S \) be the surface in \( \mathbb{R}^3 \) parameterized by
\[ \mathbf{r}(u, v) = ((2 + \cos u) \cos v, (2 + \cos u) \sin v, \sin u), \]
where \( 0 \leq u \leq 2\pi \) and \( 0 \leq v \leq 2\pi \).
(a) Sketch the surface \( S \). Do not use a calculator, laptop, phone, or any other electronic device for help.
(b) Use your parameterization in part (a) to compute the surface area of $S$.

3. Consider the surface integral
\[ \int \int_{\Sigma} z \, dS \]
where $\Sigma$ is the surface with sides $S_1$ given by the cylinder $x^2 + y^2 = 1$, $S_2$ given by the unit disk in the $xy$-plane, and $S_3$ given by the plane $z = x + 1$. Evaluate this integral as follows:

(a) Parameterize $S_1$ using $(\theta, z)$ coordinates.

(b) Evaluate the integral over the surface $S_2$ without parameterizing.

(c) Parameterize $S_3$ in (Des)cartesian coordinates and evaluate the resulting integral using polar coordinates.
4. Let $C$ be the circle in the plane with equation $x^2 + y^2 - 2x = 0$.

(a) Parameterize $C$ as follows. For each choice of a slope $t$, consider the line $L_t$ whose equation is $y = tx$. Then the intersection $L_t \cap C$ of $L_t$ and $C$ contains two points, one of which is $(0, 0)$. Find the other point of intersection, and call its $x$- and $y$-coordinates $x(t)$ and $y(t)$. Compute a formula for $\mathbf{r}(t) = (x(t), y(t))$.

(b) Suppose that $t = \frac{p}{q}$ is a rational number. Show that $x(p/q)$ and $y(p/q)$ are also rational numbers. Explain how, by clearing denominators in $x(p/q) - 1$ and $y(p/q)$, you can find a triple of integers $U, V,$ and $W$ for which $U^2 + V^2 = W^2$.

(c) Compute $\int_C \frac{1}{2} (-y, x) \cdot d\mathbf{r}$ using your parameterization above. Use Green's theorem to interpret the value of this integral.