Thursday, October 11  **  Double integral practice

1. Sketch each of the following regions in the plane. Then set up double integrals which compute the area of each using both orders of integration (one of these will usually be a better idea; we are only doing both for practice).

   (a) \( R = \{(x, y) \mid 4 \leq x^2 + y^2 \leq 100\} \)
   
   (b) \( S = \{(x, y) \mid x^2 + 4y^2 \leq 5, x \leq 0\} \)

   (c) \( T = \) the interior of the triangle with corners at \((-4, 1), (2, 3), \) and \((4, 0)\).

2. Sketch the region of integration and change the order of integration.

   (a) \( \int_0^4 \int_0^{\sqrt{x}} f(x, y) \, dy \, dx \)

   (b) \( \int_1^2 \int_0^{\ln x} f(x, y) \, dy \, dx \)

   (c) \( \int_0^1 \int_{\arctan 2}^{\pi/4} f(x, y) \, dy \, dx \)

3. Evaluate the following integrals by reversing the order of integration. You will probably want to sketch a picture of the region being integrated over.

   (a) \( \int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3 + 1} \, dx \, dy \)

   (b) \( \int_0^3 \int_{\sqrt{y}}^9 y \cos(x^2) \, dx \, dy \)

   (c) \( \int_0^1 \int_{3y}^3 e^x \, dx \, dy \)

4. Find the volume of the solid enclosed by the surfaces \( y = x^2, z = 3y, \) and \( z = 2 + y \) in \( \mathbb{R}^3 \).

5. Tomorrow we will talk about setting up double integrals in polar coordinates, but the short story is as follows:

   - In the integrand, \( x \) and \( y \) are replaced by \( r \cos \theta \) and \( r \sin \theta \).
   - The differential \( dx \, dy \) is replaced by \( r \, dr \, d\theta \).
   - The bounds of integration are changed to correspond to \( r \) and \( \theta \) bounds for the region.

Repeat parts (a) and (b) of problem 1 above using polar coordinates.