1. Let $C$ be the curve in $\mathbb{R}^2$ given by $x^3 + y^3 = 16$.
   (a) Sketch the curve $C$.
   (b) Is $C$ bounded?
   (c) Is $C$ closed?
   (d) Now consider the function $f(x, y) = e^{xy}$ on the curve $C$. Is $f$ continuous? What does the Extreme Value Theorem tell you about the existence of global min and max of $f$ on $C$?
   (e) Use Lagrange multipliers to determine the extreme values of $f$ on $C$.

2. Consider the surface $S$ given by $z^2 = x^2 + y^2$
   (a) Sketch $S$.
   (b) Use Lagrange multipliers to find the points on $S$ that are closest to $(4, 2, 0)$.
3. If the length of the diagonal of a rectangular box must be \( L \), what is the largest possible volume?

4. Use Lagrange multipliers to find the volume of the largest rectangular parallelepiped with edged parallel to the coordinate axes that can be inscribed in the ellipsoid

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.
\]