Review for Midterm 2

Determine the longest interval of existence of unique, twice differentiable solution of the IVP without finding the actual solution:

(a) \((t - 1)y'' - 3ty' + 4y = \sin t\) and \(y(-2) = 2, y'(-2) = 1\),

(b) \(t(t - 4)y'' + 3ty' + 4y = 2\) and \(y(3) = 0, y'(3) = -1\).

Find the general solution of differential equation:

(a) \(y'' + 2y' - 8y = 0\),

(b) \(y'' + 2y' + y = 0\),

(c) \(y'' + 2y' - 15y = 0\),

(d) \(4y'' + 17y' + 4y = 0\),

Use the method of order reduction to find the general solution:

(a) \(t^2y'' + 3ty' + y = 0\) and \(y_1(t) = t^{-1}\),

(b) \(xy'' - y' + 4x^3y = 0\) and \(y_1(x) = \sin x^2\),

(c) \(x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0\) and \(y_1(x) = x^{-1/2}\sin x\).

Find the general solution:

(a) \(y'' - y' - 2y = -2t + 4t^2\),

(b) \(2y'' + 3y' + y = t^2 + 3\sin t\),

(c) \(y'' + \omega_0^2y = \cos \omega_0 t\),

(d) \(y'' + y' - 6y = 12e^{3t} + 12e^{-2t}\),

(e) \(y'' + 9y = 9\sec^2 3t\),

(f) \(y'' + 4y' + 4y = \frac{e^{-2t}}{t^2}\),

(g) \((1 - t)y'' + ty' - y = 2(t - 1)^2e^{-t}\) and \(y_1(t) = e^t, y_2(t) = t\) are the solutions of homogeneous equation,
(h) \( x^2y'' - 3xy' + 4y = x^2 \ln x \) and \( y_1(x) = x^2, \ y_2(x) = x^2 \ln x \) are the solutions of homogeneous equation.

Find the solution of the IVP:

(a) \( y'' + 4y = 3 \sin 2t \) and \( y(0) = 2, \ y'(0) = -1, \)

(b) \( y'' - 2y' - 3y = 3t e^{2t} \) and \( y(0) = 1, \ y'(0) = 0, \)

(c) \( y'' + 2y' + 5y = \begin{cases} 
1, & 0 \leq t \leq \frac{\pi}{2} \\
0, & t > \frac{\pi}{2} 
\end{cases} \)

that is differentiable at \( t = \frac{\pi}{2} \) with \( y(0) = 0 \) and \( y'(0) = 0. \)

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