Review for Midterm 1

Find the general solution or solve the IVP:

(a) \( y' = xy + 3x \)
(b) \( xy' + (x + 1)y = 2xe^{-x}, \quad y(1) = y_0 \)
(c) \( ty' + 2y = \sin t, \quad t > 0 \)
(d) \( ty' - y = t^2e^{-t}, \quad t > 0 \)
(e) \( y' \sin t + y \cos t = e^t, \quad y(1) = y_0 \)

Find the general solution:

(a) \( y' = \frac{x^2}{y(1 + x^2)} \)
(b) \( xdx + ye^{-x}dy = 0, \quad y(0) = 1 \)

Solve the IVP and determine where the solution attains its:

(a) maximum value: \( y' = \frac{2 - e^x}{2 + 2y}, \quad y(0) = 0 \)
(b) minimum value: \( y' = 2(1 + x)(1 + y^2), \quad y(0) = 0 \)

Find the general solution:

(a) \( y' = \frac{x^2 + 3y^2}{2xy} \)
(b) \( y' = \frac{x^2 + xy + y^2}{x^2} \)

Solve Bernoulli–type differential equation:

(a) \( y' = \varepsilon y - \sigma y^3 \)
(b) \( y' + \frac{4}{x}y = x^3y^2, \quad y(2) = -1, \quad x > 0 \)
1. Find and classify all critical points,
2. Sketch direction field,
3. For parts (a) and (b) also find the general solution,

(a) \( y' = y(y - 1)(y - 2) \)
(b) \( y' = -k(y - 1) \), consider \( k > 0 \) and \( k < 0 \).
(c) \( y' = y^2(1 - y)^2 \)

1. Determine whether the differential equation is exact,
2. Solve if exact.

(a) \( ye^{xy} \cos 2x - 2e^{xy} \sin 2x + 2x + (xe^{xy} \cos 2x - 3)y' = 0 \)
(b) \( \frac{y}{x} + 6x + (\ln x - 2) y' = 0 \)
(c) \( x \ln y + xy + (y \ln x + xy) y' = 0 \)

Find the integrating factor and solve:

(a) \( y + (2xy - e^{-2y}) y' = 0 \)

If you are able to solve the material in this review you are well-prepared for
the midterm exam. If however you want to practice even more, feel free to
look through the Misc. Problems 1 – 31 on pages 133 – 134 in Boyce &