1. Let $S$ be the portion of the plane $x + y + z = 1$ which lies in the positive octant.

(a) Draw a picture of $S$.

**Solution.** The picture is shown below.

(b) Find a parametrization $\mathbf{r} : D \to S$, being sure to clearly indicate the domain $D$. Check your answer with the instructor.

**Solution.** One can use the parametrization $\mathbf{r}(u, v) = (u, v, 1 - u - v)$ with the domain $D$ given by $D = \{(u, v) : 0 \leq u \leq 1, 0 \leq v \leq 1 - u\}$.

(c) Use your answer in (b) to compute the area of $S$ via an integral over $D$.

**Solution.** Using the parametrization in (b), one gets

$$\mathbf{r}_u = (1, 0, -1), \quad \mathbf{r}_v = (0, 1, -1),$$

so $\mathbf{r}_u \times \mathbf{r}_v = (1, 1, 1)$, and $\|\mathbf{r}_u \times \mathbf{r}_v\| = \sqrt{3}$. Hence the area of $S$ is

$$\int \int_D dS = \int_0^1 \int_0^{1-u} \|\mathbf{r}_u \times \mathbf{r}_v\| dvdu = \frac{\sqrt{3}}{2}.$$

(d) Check your answer in (c) using only things you learned in the first few weeks of this class.

**Solution.** The picture of $S$ is a triangle with vertices $A = (1, 0, 0), B = (0, 1, 0)$ and $C = (0, 0, 1)$. Thus $\overrightarrow{AB} = (-1, 1, 0)$ and $\overrightarrow{AC} = (-1, 0, 1)$, and the area is

$$\frac{1}{2} \|\overrightarrow{AB} \times \overrightarrow{AC}\| = \frac{\sqrt{3}}{2}.$$
2. Consider the surface $S$ which is the part of $z + x^2 + y^2 = 1$ where $z \geq 0$.

(a) Draw a picture of $S$.

**Solution.** The picture is shown below.

![Surface S](image)

(b) Find a parametrization $\mathbf{r} : D \to S$. Check your answer with the instructor.

**Solution.** One can use the parametrization $\mathbf{r}(r, \theta) = (r \cos \theta, r \sin \theta, 1 - r^2)$ with the domain $0 \leq r \leq 1$, $0 \leq \theta \leq 2\pi$.

3. Let $S$ be the surface given by the following parametrization. Let $D = [-1, 1] \times [0, 2\pi]$ and define

$$\mathbf{r}(u, v) = (u \cos v, u \sin v, v).$$

(a) Consider the vertical line segment $L = \{u = 0\}$ in $D$. Describe geometrically the image of $L$ under $\mathbf{r}$.

**Solution.** The image of $u = 0$ under $\mathbf{r}$ is a line segment $(0, 0, v)$ where $0 \leq v \leq 2\pi$.

(b) Repeat for the vertical segments where $u = -1$ and $u = 1$.

**Solution.** When $u = 1$, the image $\mathbf{r}(1, v) = (\cos v, \sin v, v)$ is a helix with $0 \leq v \leq 2\pi$, and so is $u = -1$. Thus the images of $u = 1$ and $u = -1$ form the double helix.

(c) Use your answers in (a) and (b) to make a sketch of $S$.

**Solution.** The picture is shown below.
4. Consider the ellipsoid $E$ given by $\frac{x^2}{9} + \frac{y^2}{4} + z^2 = 1$.

(a) Draw a picture of $E$.

**Solution.** The picture is shown below.

(b) Find a parametrization of $E$. Hint: Find a transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ which takes the unit sphere $S$ to $E$, and combine that with our existing parametrization of the plain sphere $S$.

**Solution.** One can use the following parametrization

$$\mathbf{r}(\theta, \phi) = (3 \sin \theta \cos \phi, 2 \sin \theta \sin \phi, \cos \theta)$$

with the domain $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$. 