Thursday, August 27  

Parametric curves defined using vector arithmetic.

1. Let $f(x) = x^2 + x - 2$.

   (a) Graph the equation $y = f(x)$. (By hand, then check with a calculator if you want.)

   (b) Find the slope and equation of the tangent line to $y = f(x)$ when $x = 2$. Draw the tangent line on your picture.

   (c) Draw a vector in $\mathbb{R}^2$ that describes the direction of the line. Find a numeric representation of your vector.

2. Consider the curve given parametrically by

   \[
   \begin{align*}
   x(t) &= t \\
   y(t) &= t^2 + t - 2
   \end{align*}
   \]

   for $0 \leq t < 4$.

   (a) Sketch the curve. How does this graph differ from your graph in Problem 1(a)?

   (b) Consider the vectors formed by the pair $(x(t), y(t))$. Anchoring the vectors at the origin, sketch on your graph the vectors at time $t = 0, 1, 2, 3$.

   (c) Now consider the vectors formed by $(x'(t), y'(t))$. Evaluate $(x'(t), y'(t))$ at time $t = 2$, what does the vector represent? Hint: Graph it on the curve at the point $(x(2), y(2))$.

   (d) Imagine that the curve is the path of a moving particle. What is the speed of the particle when $t = 2$?

3. (a) Sketch the vector emanating from the origin ending at the point $(-5, 2)$ in $\mathbb{R}^2$.

   (b) On the same graph and using the “head-to-tail” geometric addition method, draw the vector $(-5, 2) + (3, -1)$.

   (c) Do the same for $(-5, 2) + 2(3, -1)$.

   (d) Do the same for $(-5, 2) + (-1)(3, -1)$.

   (e) If we allow the scalar multiplying the vector $(3, -1)$ to vary, what geometric object is described by the parametric equation $(-5, 2) + t(3, -1)$ for all $t$?

4. Consider the set of points in $\mathbb{R}^3$ defined by the parametric equation

   \[ I(t) = (-5 + 2t, 2 + 3t, 1 - t) \quad \text{for all } t. \]

   (a) Using the properties of vector arithmetic, factor $I(t)$ into the form $p + tv$ where $p$ and $v$ are vectors in $\mathbb{R}^3$.

   (b) Using the factored form (and your technique from Problem 3) sketch this object in $\mathbb{R}^3$. Geometrically, what does this parametric equation describe?

   (c) Why is the vector $v$ in your factored form referred to as the direction vector?

5. Let $a = (-\sqrt{3}, 0, -1, 0)$ and $b = (1, 1, 0, 1)$ be vectors in $\mathbb{R}^4$.

   (a) Find the distance between the points $(-\sqrt{3}, 0, -1, 0)$ and $(1, 1, 0, 1)$.

   (b) Find the angle between $a$ and $b$. 
