Your name: ___________________________  Your NetID: __________________________

- No notes, books, or electronics out or hats or sunglasses on during the exam.
- You must show your work on all questions. This means show the work that an average Math 241 student would reasonably require.
- Do not guess on multiple choice problems—you will receive one point on any multiple choice problem left blank.

Mark your discussion Section in the table below:

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1. (7 points) Rewrite the triple integral

\[ \int_{0}^{2} \int_{x}^{2} \int_{1}^{3-y} z^2 \, dz \, dy \, dx \]

using the different order of integration specified below.

\[ \begin{cases} 0 \leq x \leq 2 \\ x \leq y \leq 2 \\ 1 \leq z \leq 3-y \end{cases} \quad \text{is the same as} \quad \begin{cases} 0 \leq x \leq y \leq 2 \\ 1 \leq z \leq 3-y \end{cases} \]

\[ \int_{0}^{2} \int_{x}^{2} \int_{1}^{3-y} z^2 \, dz \, dy \, dx = \int \int \int z^2 \, dx \, dz \, dy \]

\[ \begin{array}{ccc} 2 & 3-y & y \\ 0 & 1 & 0 \end{array} \]
2. (4 points) The volume of a region $R$ is calculated as a triple integral in spherical coordinates as

$$\iiint_{R} dV = \int_{1}^{2} \int_{\pi/2}^{\pi/2} \int_{\pi/2}^{\pi} \rho^2 \sin \phi \ d\theta \ d\phi \ d\rho.$$

Circle the picture of the region $R$. 
3. (8 points) Consider the vector field \( \mathbf{F}(x, y, z) = (xz, e^y - yz, \cos x) \).

(a) Find \( \text{curl} \, \mathbf{F} \).

\[
\text{curl} \, \mathbf{F} = \begin{vmatrix}
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
xz & e^y - yz & \cos x \\
\end{vmatrix} = (\frac{\partial}{\partial y} (\cos x) - \frac{\partial}{\partial z} (e^y - yz)) \mathbf{i} \\
-(\frac{\partial}{\partial x} (xz) - \frac{\partial}{\partial z} (x^2)) \mathbf{j} + (\frac{\partial}{\partial x} (e^y yz) - \frac{\partial}{\partial y} (x^2)) \mathbf{k} \\
= (-e^y + y) \mathbf{i} + (\sin x + x) \mathbf{j}
\]

(b) Find \( \text{div} \, \mathbf{F} \).

\[
\text{div} \, \mathbf{F} = \frac{\partial}{\partial x} (xz) + \frac{\partial}{\partial y} (e^y - yz) + \frac{\partial}{\partial z} (\cos x) = 2 - 2 = 0
\]

(a) Does there exist a function \( f \) with \( \nabla f = \mathbf{F} \)? Circle the correct response.

Yes  \( \boxed{\text{No}} \)  We do not have enough information.

If \( \mathbf{F} \) were conservative, then we'd have \( \text{curl} \, \mathbf{F} = 0 \). But \( \text{curl} \, \mathbf{F} \neq 0 \),

thus \( \mathbf{F} \) is not conservative.
4. (4 points) Let \( B \) be the region in the plane bounded by the smooth, simple closed curve \( C \) drawn below, where \( C \) is oriented counterclockwise.

Which of the integrals below computes the area of \( B \)? Circle your response.

By Green's Theorem
\[
\int_C x \, dx = \int_C x \, dx + y \, dy = \iint_B \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dA = 0
\]
\[
\frac{1}{2} \int_C y \, dx + x \, dy = \frac{1}{2} \int_B \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dA = 0
\]
\[
\int_C 2xe^{y} \, dx + x(1+xe^{y}) \, dy = \int_B \left( \frac{\partial (x+xe^{y})}{\partial x} - \frac{\partial (2xe^{y})}{\partial y} \right) \, dA = \int_B (1+2xe^{y}-2xe^{y}) \, dA = \int_B 1 \, dA = \text{Area}(B)
\]

\[
\int_C 2xe^{y} \, dx + x(1+xe^{y}) \, dy \quad \frac{1}{2} \int_C y \, dx + x \, dy \quad \int_C x \, dx \quad \text{None of these}
\]

5. (4 points) The vector field \( F \) on \( \mathbb{R}^3 \)

is shown in the \( xy \)-plane and looks the same in all other horizontal planes.

Circle the best completion of the sentence below.

The divergence of \( F \)...

...is positive ...

...is negative.

...points up.

...points left.

...is zero.
6. (8 points) Consider the region \( R \) bound by a parallelogram shown at the right.

(a) Circle the transformation \( T: \mathbb{R}^2 \to \mathbb{R}^2 \) sending the unit square \([0, 1] \times [0, 1]\) onto the region \( R \).

All five transformations below are of the form \( T(u,v) = (au + bv, cu + dv) \), so they satisfy \( T(0, 0) = (0,0) \). We need either
\[
\begin{align*}
T(0,1) &= (2,4) = (b,d) \\
T(1,0) &= (6,1) = (a,c) \\
T(1,1) &= (8,5) \\
T(u,v) &= (2u+6v, 4u+v)
\end{align*}
\]

\[
T(u, v) =\begin{cases}
(6u + v, 2u + 4v) & \text{if } T(0,0) = (0,0) \\
(6u - 2v, u + 4v) & \text{if } T(0,0) = (0,0) \\
(6u + v, 4u + 2v) & \text{if } T(0,0) = (0,0) \\
(6u + 2v, 4u + v) & \text{if } T(0,0) = (0,0) \\
(6u + 4v, u + 2v) & \text{if } T(0,0) = (0,0)
\end{cases}
\]

(b) Suppose \( D \) is the triangle with vertices \((6, 1), (2, 4), (8, 5)\). Change coordinates using the transformation \( T \) found above to calculate the integral. Circle the correct answer. If you left (a) blank, clearly specify one of the choices for \( T \) here and calculate using that, assuming it takes the unit square to \( R \).

\[
T(u,v) = (6u + 2v, u + 4v) = (x, y)
\]

\[
\frac{\partial (x,y)}{\partial (u,v)} = \begin{vmatrix} 6 & 2 \\ 1 & 4 \end{vmatrix} = 22
\]

\[
\iint_D (x-y) dA = \iint_0^1 \left( \frac{1}{5}\left(6u + 2v - u - 4v\right) \right) 22 \, dv
\]

\[
\iint_D x - y \, dA = \int_0^1 \int_0^1 F(u,v) \, dv \, du
\]

\[
F(u,v) = \begin{cases}
88u - 66v & \text{if } u = 0 \\
110u - 44v & \text{if } u = 1 \\
16u - 8v & \text{if } u = 0 \\
4u + 2v & \text{if } u = 1 \\
40u + 16v & \text{if } u = 1
\end{cases}
\]
7. (7 points) Let S be the surface parameterized by \( \mathbf{r}(u, v) = (v^2 - u^2, u, v) \) with \( \{(u, v) \mid -2 \leq u \leq 2 \text{ and } -2 \leq v \leq 2\} \)

(a) Circle the picture of S. 

(b) The surface area of S is calculated by the integral \( A(S) = \int_{-2}^{2} \int_{-2}^{2} F(u, v) \, du \, dv \).

Circle the correct expression for \( F(u, v) \).

\[
\begin{align*}
\mathbf{r}_u &= \langle -2u, 1, 0 \rangle, \quad \mathbf{r}_v = \langle 2v, 0, 1 \rangle \\
\mathbf{r}_u \times \mathbf{r}_v &= \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
-2u & 1 & 0 \\
2v & 0 & 1 \\
\end{vmatrix} = \mathbf{i}(2) - \mathbf{j}(-2) + \mathbf{k}(0) = 2 \mathbf{i} + 2 \mathbf{j}
\end{align*}
\]

\( F(u, v) \, du \, dv = |\mathbf{r}_u \times \mathbf{r}_v| \, du \, dv = \sqrt{1 + 4u^2 + 4v^2} \, du \, dv 
\]

(c) Circle the correct response:

The integral \( \iint_S x^2 (y - 5) \, dS \) is \( \text{positive} \)\( \text{negative} \) \( \text{zero} \)

because \( y - 5 = u - 5 \leq 2 - 5 < 0 \) and \( x^2 (y - 5) < 0 \) on \( S \), except on the "thin" set \( \{(0, u, \pm u) \mid -\frac{5}{2} \leq u \leq \frac{5}{2}\} \).
8. (8 points) Find a parameterization \( \mathbf{r}(u, v) \) for each of the surfaces described below. Use \( u, v \) as your parameters, and specify the domain \( D \) of the parameterization.

**Important:** The domain \( D \) must be a rectangle.

(a) The part of the surface \( z = (1 - x^2)(4 - y^2) \) where \( z \geq 0 \) and \( -2 \leq y \leq 2 \).

If \( -2 < y < 2 \) then \( y^2 < 4 \) and \( 4 - y^2 > 0 \). Hence \( z > 0 \) amounts to
\[ 1 - x^2 \geq 0, \text{ that is } -1 \leq x \leq 1 \] and \( D = [-1, 1] \times [-2, 2] \).
\[ \mathbf{r}(u, v) = (u, v, (1-u^2)(4-v^2)), \quad -1 \leq u \leq 1, \quad -2 \leq v \leq 2. \]

\[ \mathbf{r}(u, v) = \begin{pmatrix} u \\ v \\ (1-u^2)(4-v^2) \end{pmatrix} \]
\[ D = \left\{ (u, v) \mid -1 \leq u \leq 1 \text{ and } -2 \leq v \leq 2 \right\} \]

(b) The part of the cylinder \( x^2 + z^2 = 9 \) that lies between the planes \( y = 0 \) and \( y = 1 \), and for which \( z \geq 0 \).

\[ \begin{aligned}
  x &= 3 \cos u \\
  z &= 3 \sin u \\
  y &= v
\end{aligned} \]
\[ 0 \leq u \leq 2\pi \text{ and } z \geq 0 \text{ implies } 0 \leq u \leq \pi \]
\[ 0 \leq v \leq 1 \]

\[ \mathbf{r}(u, v) = \begin{pmatrix} 3 \cos u \\ v \\ 3 \sin u \end{pmatrix} \]
\[ D = \left\{ (u, v) \mid 0 \leq u \leq \pi \text{ and } 0 \leq v \leq 1 \right\} \]