Name: Solutions

The goal of this worksheet is to better understand the last-diminisher method for 3+ people. Suppose there are 4 people: Alex, Bill, Chris, and Derek. Together they paid $16 for an extra large pizza which is half veggie half pepperoni. The pizza is cut into 8 pieces (4 veggie and 4 pepperoni).

- Alex and Derek like veggie pizza and pepperoni pizza the same amount.
- Bill likes veggie pizza three times as much as he likes pepperoni pizza.
- Chris likes pepperoni pizza three times as much as he likes veggie pizza.

\[ \text{Alex + Derek} \quad \frac{$16}{8} = \frac{$2}{1} \text{ for any slice} \]
\[ \text{Bill: let } x = \text{Bill's value of 1 pepperoni slice.} \]
\[ \Rightarrow 4(3x) + 4(x) = $16 \]
\[ 12x + 4x = $16 \Rightarrow x = $1 \text{ for pep. slice} \]
\[ x = $3 \text{ for veggie slice} \]

\[ \text{symmetric argument for Chris} \]

Calculate how much each slice of pizza is worth to each of the 4 people.

<table>
<thead>
<tr>
<th></th>
<th>1 slice of veggie</th>
<th>1 slice of pepperoni</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alex</td>
<td>$2</td>
<td>$2</td>
</tr>
<tr>
<td>Bill</td>
<td>$3</td>
<td>$1</td>
</tr>
<tr>
<td>Chris</td>
<td>$1</td>
<td>$3</td>
</tr>
<tr>
<td>Derek</td>
<td>$2</td>
<td>$2</td>
</tr>
</tbody>
</table>

Now let's use the last-diminisher method to split the pizza.

1. Alex takes 2 slices of pepperoni pizza and passes it down. (worth 2 ($2) = $4)
   a. Bill's response: worth 2 ($1) = $2
      so Bill passes.
   b. Chris' response: worth 2 ($3) = $6 > $4. Note: \( \frac{1}{3} \) a slice of pep.
      worth $1. So Chris trims the slice and passes 1+\( \frac{1}{3} \) slice.
   c. Derek's response: 1+\( \frac{1}{3} \) slices of pep. = $2.67 \Rightarrow Derek passes.
   d. Who leaves during this round, and what is their share of the pizza?
      Chris leaves w/ 1+\( \frac{1}{3} \) slice of pepperoni pizza.
2. Alex takes 2 slices of veggie pizza and passes it down.
   a. Bill's response:
      \[ \text{worth} 2(\$2) = \$4 \]
      \[ \text{worth} 2(\$3) = \$6 > \$4 \], so he trims off a piece, passes down \( 1 + \frac{1}{3} \) pieces of veggie pizza.
   b. Derek's response:
      \[ 1 + \frac{1}{3} \text{ veggie pizza } \text{ worth} \approx \$2.67, \text{ so Derek passes}. \]
   c. Who leaves during this round, and what is their share of the pizza?
      Bill leaves w/ \( 1 + \frac{1}{3} \) slices of veggie.

3. Alex takes 1 slice of veggie pizza and 1 slice of pepperoni pizza and passes it down.
   a. Derek's response:
      \[ \text{worth} \$2 + \$2 = \$4 \], He passes.
   b. Who leaves during this round, and what is their share of the pizza?
      Since Alex was the (only) person to cut this slice, he takes it. ('veggie, 1 pepperoni')

4. What is the final distribution of the pizza?
   Chris = \( 1 + \frac{1}{3} \) pep,  \( B.i.l = 1 + \frac{1}{3} \) veggie
   Alex = 1 pep, 1 veggie  Derek = \( 1 + \frac{2}{3} \) veggie, \( 1 + \frac{2}{3} \) pepper

5. How much does each person value their share of the pizza?
   Chris, Bill, Alex: \$4.
   Derek: \[ 2(1 + \frac{2}{3} + 1 + \frac{2}{3}) \approx \$6.67 \]

6. Is this division proportional? Envy-free?
   Proportional \( \checkmark \) everyone believes they got at least \$4 worth.
   Envy-free - NO, everyone envies Derek!