Suppose you are playing a game with a friend. At the count of three, you and your friend each reveal a coin. If both coins are heads, then your friend pays you $5. If both coins are tails, then your friend pays you $1. If there is 1 heads and 1 tails, you pay your friend $3.

Fill in the payoff matrix in terms of how much money you gain or lose (you = rows, friend = columns).

<table>
<thead>
<tr>
<th></th>
<th>HEADS</th>
<th>TAILS</th>
</tr>
</thead>
<tbody>
<tr>
<td>HEADS</td>
<td>(5, -5)</td>
<td>(3, 3)</td>
</tr>
<tr>
<td>TAILS</td>
<td>(-3, 3)</td>
<td>(1, -1)</td>
</tr>
</tbody>
</table>

You and your friend both decide to play with some randomness. You decide to choose heads with probability p and tails with probability 1-p. Your friend will play heads with probability q and tails with probability 1-q. We want to find optimal values for p and q.

To choose your value of p you should consider the following:

- If p is too high, i.e., if you play heads too often, your friend will counter your strategy by choosing tails very often.
- If p is too low, i.e., if you play tails too often, your friend will counter your strategy by choosing heads very often.
- So you should choose the probability p so that your friend has no advantage choosing heads or choosing tails. That is, you should choose p so that your friend has the same expected payoff if they play heads versus if they play tails.

Let \( F_H \) be the payoff for your friend if they play heads and let \( F_T \) be their payoff for playing tails.

Calculate the expected values of \( F_H \) and \( F_T \):

\[
E(F_H) = (-5)p + 3(1-p) = -5p + 3 - 3p = -8p + 3
\]

\[
E(F_T) = 3p + (-1)(1-p) = 3p - 1 + p = 4p - 1.
\]

Now solve for p so that these expected values are equal. What is your optimal strategy?

\[-8p + 3 = 4p - 1\]

\[4 = 12p \rightarrow p = \frac{1}{3}\]

Now consider your friend’s point of view. They play heads with probability q and tails with probability 1-q. Solve for an optimal value of q.

Let \( Y_H \) and \( Y_T \) be your payoffs for playing \( H \) and \( T \) respectively.

\[
E(Y_H) = (5)q + (-3)(1-q) = 5q - 3 + 3q = 8q - 3
\]

\[
E(Y_T) = (-3)q + (1)(1-q) = -3q + 1 - q = -4q + 1
\]

\[8q - 3 = -4q + 1\]

\[12q = 4 \rightarrow q = \frac{1}{3}\]

So friend’s strategy is \( (\frac{1}{3} H, \frac{2}{3} T) \). Together, Nash Eq. : both players play \( \frac{1}{3} H, \frac{2}{3} T \).
1. Find the pure Nash equilibria of the following games.
   a. 
   \[
   \begin{array}{|c|c|c|}
   \hline
   & \text{Coop.} & \text{Defect} \\
   \hline
   \text{Coop.} & (5,3) & (1,1) \\
   \hline
   \text{Defect} & (0,0) & (3,5) \\
   \hline
   \end{array}
   \]
   \[
   (\text{Coop, Coop}), (\text{Defect, Defect})
   \]
   b. 
   \[
   \begin{array}{|c|c|c|}
   \hline
   & \text{Coop.} & \text{Defect} \\
   \hline
   \text{Coop.} & (1,1) & (2,0) \\
   \hline
   \text{Defect} & (-1,2) & (3,3) \\
   \hline
   \end{array}
   \]
   \[
   (\text{coop, Coop}), (\text{Defect, Defect})
   \]

2. Alice and Bob have agreed to go on a date tomorrow at noon. However, both of them forgot what the meeting place was and have no way to contact each other prior to the date. They remember that they were deciding between going to a movie or going to a concert. Alice would rather go to the movie, and Bob would rather go to the concert. They must both decide independently where they will go and hope that the other shows up at the same place. The following is a payoff table representing each person’s happiness per scenario (Alice = rows, Bob = columns).

   \[
   \begin{array}{|c|c|}
   \hline
   & \text{Movie} & \text{Concert} \\
   \hline
   \text{Movie} & (2,1) & (0,0) \\
   \hline
   \text{Concert} & (0,0) & (1,2) \\
   \hline
   \end{array}
   \]
   a. Find the pure Nash equilibria of the game.
      Both go to movie, both go to concert.
   b. Using the previous side of the worksheet as a guide, find the mixed Nash equilibria of the game.
      Let Alice go to movie w/ prob. \( p \), concert w/ prob. \( 1-p \).
      Let Bob go to movie w/ prob. \( q \), concert w/ prob. \( 1-q \).
      Alice’s payoffs: \( A_m, A_c \).
      \[
      E(A_m) = 2q + 0(1-q) = 2q, \quad E(A_c) = 0q + 1(1-q) = 1-q.
      \]
      \[
      2q = 1-q \quad \Rightarrow \quad q = \frac{1}{3} \quad \Rightarrow \quad 1-q = \frac{2}{3}.
      \]
      Bob’s payoffs: \( B_m, B_c \).
      \[
      E(B_m) = (1)p + 0(1-p) = p, \quad E(B_c) = 0p + 2(1-p) = 2-2p.
      \]
      \[
      p = 2-2p \quad \Rightarrow \quad p = \frac{2}{3} \quad \Rightarrow \quad 1-p = \frac{1}{3}.
      \]
      Let \( B_m, B_c \) be payoffs for Bob if he goes to movie or concert, respectively.