Note: I put an extra question on this practice exam for more practice. The actual test may be shorter.

• You have 50 minutes.

• You may use a single (one-sided) page of hand-written notes prepared by you and approved by the instructor in advance.

• There are extra blank sheets of paper attached to the end of the exam that you may use for scratch work.

• You may use a calculator.

• No otherwise cheating!
1. (20 points total) In this game of lottery, a player must guess (in the correct order) 3 numbers, where each number can be from 0-9. A player that correctly guesses all 3 numbers wins the big jackpot. Each player is only allowed to play one time.

(a) (2 points) What is the total number of possible combinations?

(b) (3 points) Let $A$ be the event that a player guesses the first number correctly. What is the probability of event $A$, i.e., $P(A)$?

(c) (3 points) Let $B$ be the event that a player wins the big jackpot. What is $P(B)$?

(d) (2 points) State the formula for the conditional probability $P(B|A)$.

(e) (5 points) Define event $A$ as in part (b) and event $B$ as in part (c). Calculate $P(B|A)$.

(f) (1 points) Are events $A$ and $B$ independent? (No explanation is necessary.)

(g) (4 points) A player can win a small jackpot if they guess the first two numbers correctly and the last number incorrectly. What is the probability of winning a small jackpot?
2. (10 points total) Suppose you flip a fair coin 4 times.

(a) (1 point) What is the total number of possible outcomes of flipping the coin 4 times?

(b) (2 point) What is the probability of flipping 4 tails?

(c) (3 points) Let $A$ be the event that you flip at least 1 heads. Describe, in words, what the event $\overline{A}$ describes. Write all of the possible outcomes belonging to $\overline{A}$.

(d) (4 points) Let $A$ be the event defined in part (c). What is $P(A)$? (Hint: use part (c).)
3. (10 points total) Match the voting methods with their descriptions. (2 points each)

(i) Condorcet  (ii) Borda Count  (iii) Instant Run-off (Hare system)
(iv) Plurality  (v) Sequential pairwise

___ The winner is the candidate who would beat every other candidate in a head-to-head election.

___ The winner is the candidate with the most first place votes.

___ Every round, the candidate(s) with the fewest first place votes is eliminated until only one remains.

___ An agenda (i.e., an ordering of the candidates) is specified. The first and second candidate face off. The loser is eliminated and the winner goes on to face the third candidate. We continue this until only one candidate remains.

___ For each ballot, the last place candidate gets 0 points, the second-to-last place candidate gets 1 point, etc. A candidate’s overall score is the sum of their scores over all ballots. The winner is the candidate with the highest score.
4. (16 points total) Consider the election with 17 voters below.

<table>
<thead>
<tr>
<th></th>
<th>5 votes</th>
<th>4 votes</th>
<th>3 votes</th>
<th>1 votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>C</td>
<td>B</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>B</td>
<td>C</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>A</td>
<td>C</td>
<td></td>
</tr>
</tbody>
</table>

(a) (5 points) Use the Instant Run-off (Hare) method to find a winner of the election. (Note: if a candidate gets 0 first place votes, they get eliminated first.)

(b) (3 points) If the last two voters change their votes to A,B,D,C, find the winner using the Instant Run-off method.

(c) (2 points) What does this tell you about the Instant Run-off method?

(d) (6 points) Calculate the Borda Scores of all the candidates in the original election.
5. (20 points total) Jacob and Betty purchased a fruit basket consisting of 8 apples, 9 oranges, and 8 peaches. Together, they paid $10.00 for the fruit basket.

- Jacob is allergic to oranges (so they have no value to him), and he likes peaches four times as much as he likes apples.
- Betty likes apples, oranges, and peaches equally.

(a) (6 points total) Calculate the value (as a dollar amount) of each fruit to Jacob and Betty respectively.

(b) (3 points) Suppose Jacob received all the peaches, and Betty received all of the apples and oranges. Is this a proportional division?

(c) (3 points) Show that the division in part (b) is envy-free.
(d) (5 points) Now let Jacob and Betty use the Divide-and-Choose method to divide the fruit. Let Jacob divide the fruits into two piles that he considers equal (i.e., worth $5.00). Write down such a division.

(e) (3 points) Which pile would Betty rather take? Why?
6. (20 points total) Coaches for 3 different teams are trying to draft new players. They decide to use the **Lone-Divider** method to make a proportional division. There are 6 players who we will call Player 1, Player 2, and so on. Each coach gets 24 “points” to distribute to the players which indicate how much they value each player (more points mean higher value). Below is a chart with the coaches’ valuations.

<table>
<thead>
<tr>
<th></th>
<th>Player 1</th>
<th>Player 2</th>
<th>Player 3</th>
<th>Player 4</th>
<th>Player 5</th>
<th>Player 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coach A</td>
<td>8</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Coach B</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Coach C</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

(You should think of this as similar to the cake-cutting activity on the back of WS 7.)

(a) (4 points) First have Coach A divide the players into 3 groups that he believes have equal value (i.e., each group is worth $24/3 = 8$ points).

(b) (2 points) Each other coach calls a group *acceptable* if they value the group at at least 8 points. Which of the groups made by Coach A does Coach B find acceptable? Which of the groups does Coach C find acceptable?

(c) (4 points) For simplicity, let Coach A keep the group with the fewest players. Then let Coach B and C use Divide-and-Choose on the rest of the players. Write a division of the remaining players into two groups that Coach B considers equal in value.

(d) (3 points) Which group would Coach C rather keep?
(e) (3 points) Write the final distribution of players to coaches.

Coach A: ________________________________

Coach B: ________________________________

Coach C: ________________________________

(f) (2 points) Is this division proportional? (No explanation is required.)

(g) (2 points) Is this division envy-free? If not, who envies whom?
7. (14 points total/ 2 points each) Circle true or false.

(a) (TRUE / FALSE) The Selfridge–Conway method is envy-free and proportional.

(b) (TRUE / FALSE) The Last-Diminisher method is envy-free and proportional.

(c) (TRUE / FALSE) In the Borda Count voting system, the candidate with the most first place votes is always the winner.

(d) (TRUE / FALSE) When rolling a (fair) 6-sided die, the probability of rolling an even number is the same as the probability of rolling an odd number.

(e) (TRUE / FALSE) For any events $A$ and $B$, $P(A \cap B) = P(A)P(B)$.

(f) (TRUE / FALSE) For any event $A$, $A$ and $\overline{A}$ are disjoint events.

(g) (TRUE / FALSE) The Condorcet voting system always chooses a single winner.